A Realistic Baryon-Baryon Interaction in the SU_6 Quark Model and its Applications to Few-Baryon Systems

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The recent quark-model baryon-baryon interaction by the Kyoto-Niigata group is applied to the three-nucleon, $2\alpha\Lambda$ and $2\Lambda\alpha$ systems, in which a new three-cluster Faddeev formalism, using the 2-cluster resonating-group method (RGM) kernel, is developed for the exact treatment of the Pauli forbidden states between clusters.

1. INTRODUCTION

The QCD-inspired spin-flavor SU_6 quark model for the baryon-baryon interaction, proposed by the Kyoto-Niigata group, is a unified model for the complete baryon octet ($B_8 = N$, Λ, Σ and Ξ), which has achieved very accurate descriptions of the NN and YN interactions. [1–3] In particular, the nucleon-nucleon (NN) interaction of the most recent model fss2 [2] is accurate enough to compare with the modern realistic meson-exchange models. These quark-model interactions can be used for realistic calculations of few-baryon and few-cluster systems, once an appropriate three-body equation is developed for the pairwise interactions described by the RGM kernel.

2. 3-CLUSTER FADDEEV FORMALISM USING THE 2-CLUSTER RGM KERNEL

The desired 3-cluster equation should be able to deal with the non-locality and the energy-dependence intrinsically involved in the quark-exchange RGM kernel. Furthermore, the quark-model description of the hyperon nucleon (YN) and the hyperon hyperon (YY) interactions in the full coupled-channel formalism sometimes involves the Pauli forbidden state, which excludes the most compact spatial configuration, resulting

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in the strongly repulsive nature of the interaction in some particular channels. We have recently achieved a new 3-cluster formalism which uses two-cluster RGM kernel explicitly. [4] The equation entirely eliminates 3-cluster redundant components by the orthogonality of the total wave function to the pairwise two-cluster Pauli-forbidden states. The explicit energy dependence inherent in the exchange RGM kernel is self-consistently determined. This equation is entirely equivalent to the Faddeev equation which uses a modified singularity-free T-matrix (which we call the RGM T-matrix) constructed from the two-cluster RGM kernel. We first applied this formalism to a 3-dineutron system and the 3α system, and obtained a complete agreement between the Faddeev calculation and the variational calculation which uses the translationally invariant harmonic-oscillator (h.o.) basis. [4,5]

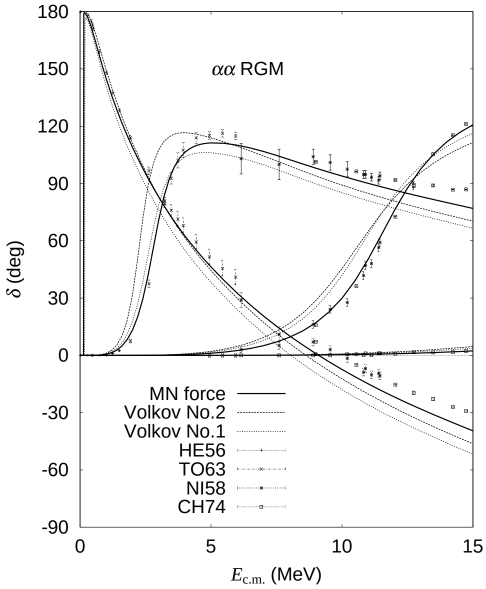

Figure 1. 2α phase shifts calculated by the MN force (thick solid curves) and the Volkov forces (others).

For the 3α system, the input is the 3-range Minnesota force (MN force) with the exchange mixture $u = 0.946869$, and the h.o. width parameter, $\nu = 0.257$ fm^{-2}, is used for the $(0s)^4$ α-clusters. The 2α phase shifts are nicely reproduced in the 2α RGM, as shown in Fig. 1. We find that the 3α ground-state energy obatined by solving the present 3α Faddeev equation is only about 1.5 MeV higher than that of the full microscopic 3α RGM calculation. [5]

3. 3-NUCLEON BOUND STATE

The present 3-cluster RGM formalism was applied to the Faddeev calculation of the 3-nucleon bound state, employing the off-shell T-matrices which are derived from the non-local and energy-dependent RGM kernel for our quark-model NN potentials , fss2 and FSS. [6] The model fss2 yields the ground-state energy $E(^3\mathrm{H}) = -8.519$ MeV in the 50 channel calculation, when the np interaction is employed for any NN pairs. [7] The charge rms radii for ^{3}H and ^{3}He are also correctly reproduced. These results are the closest to the experiments among many results obtained by Faddeev calculations, employing modern realistic NN interactions. A characteristic description of the short range correlations in the quark model is essential to reproduce the large binding energy and the correct size of the three-nucleon bound state without reducing the D-sate probability of the deuteron.

4. $2\alpha\Lambda$ FADDEEV CALCULATION FOR $^9_\Lambda$Be

The formalism is now applied to the $2\alpha\Lambda$ Faddeev calculation for $^9_\Lambda$Be, using the 2α RGM kernel and the $\Lambda\alpha$ folding potential for the ΛN effective interaction. The effective

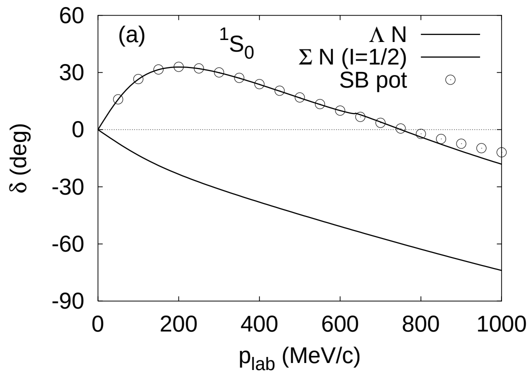

Figure 2. ΛN $^1S_0$ phase shifts by fss2 (solid curve) and by SB potential (Circles).

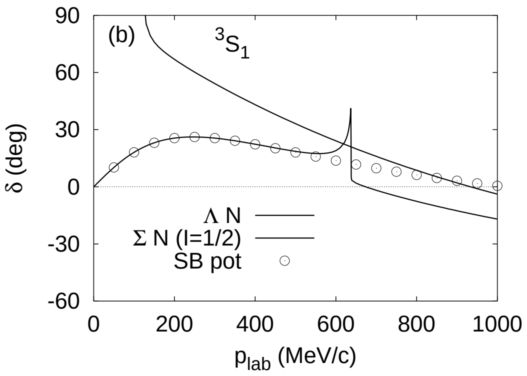

Figure 3. ΛN $^3S_1$ phase shifts by fss2 (solid curve) and by SB potential (Circles).

ΛN interaction, denoted by SB (Sparenberg-Baye potential) in Table 1, is constructed from the $^1S_0$ and $^3S_1$ phase shifts predicted by the YN sector of the model fss2 [3], by using an inversion method based on supersymmetric quantum mechanics. These are simple 2-range Gaussian potentials which reproduce the low-energy behaviour of the ΛN (Figs. 2 and 3) and $\Lambda\Lambda$ (Fig. 4) phase shifts obtained by the full coupled-channel calculations. Since any central and single-channel effective ΛN force leads to the well-known overbinding problem of $^5_\Lambda$He [8] by about 2 MeV (in the present case, it is 1.63 MeV), the attractive part of the $^3S_1$ ΛN potential is adjusted to reproduce the correct binding energy $E^{\rm exp}(^5_\Lambda{\rm He}) = -3.12 \pm 0.02$ MeV. The odd-state ΛN interaction is assumed to be zero. The partial waves up to $\lambda_{\rm Max} = \ell_{1\rm Max} = 6$ are included both in the 2α and $\Lambda\alpha$ channels. The direct and exchange Coulomb kernel between two α-clusters is introduced at

Table 1
The ground-state energy $E_{\rm gr}(0^+)$ and the 2^+ excitation energy $E_{\rm x}(2^+)$ in MeV, calculated by solving the Faddeev equation for the $2\alpha\Lambda$ system. ΔE in the last column is a simple estimation of the ℓs splitting for the $5/2^+$ and $3/2^+$ excited states, using the $\boldsymbol{P}$=0 Wigner tarnsform of the fss2 and FSS ΛN LS interaction. "$+\sigma$" in fss2 implies the σ-meson contribution, which is not yet calculated.

$V_{\Lambda N}$	$E_{\rm gr}(0^+)$ (MeV)			$E_{\rm x}(2^+)$	ΔE (keV)	
	ours	Hiyama	diff.	(MeV)	fss2	FSS
SB	-6.837	–	–	2.915	$103+\sigma$	164
NS	-6.742	-6.81	0.07	2.916	$97+\sigma$	154
ND	-7.483	-7.57	0.09	2.935	$106+\sigma$	169
NF	-6.906	-7.00	0.09	2.930	$86+\sigma$	138
JA	-6.677	-6.76	0.08	2.919	$86+\sigma$	138
JB	-6.474	-6.55	0.08	2.911	$85+\sigma$	136
Exp't	-6.62 ± 0.04			3.029(3)/3.060(3)	43 ± 5	

the nucleon level with the cut-off radius, $R_C = 14$ fm (central case) or 10 fm (ℓs included). Table 1 shows the ground-state (0^+) and the excited-state (2^+) energies of ${}^{9}_{\Lambda}$Be, predicted by the SB and the other various ΛN potentials used by Hiyama *et al.* [9]. In the present calculations using only the central force, the SB potential with the pure Serber character can reproduce the ground-state and the excited-state energies within the accuracy of 100 - 200 keV. Table 1 also shows a simple estimation of the ℓs splitting for the $5/2^+$ and $3/2^+$ excited states, due to the spin-orbit interaction predicted by fss2 and FSS.

5. $2\Lambda\alpha$ FADDEEV CALCULATION FOR ${}^{6}_{\Lambda\Lambda}$He

Next, we use the $\Lambda\alpha$ T-matrix, used in the $2\alpha\Lambda$ Faddeev calculation, to calculate the ground-state energy of ${}^{6}_{\Lambda\Lambda}$He. The full coupled-channel T-matrices of fss2 and FSS with the strangeness $S=-2$ and the isospin $I=0$ [3] are employed for the $\Lambda\Lambda$ RGM T-matrix. Table 2 shows the $\Delta B_{\Lambda\Lambda}$ values (in MeV) defined by $\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) - 2B_{\Lambda}({}^{5}_{\Lambda}\mathrm{He})$. $V_{\Lambda\Lambda}$(Hiyama) uses the 3-range Gaussian potential in [9]. We find that the Hiyama's 3-range Gaussian $\Lambda\Lambda$ potential and our Faddeev calculation using FSS yield very similar results with the large $\Delta B_{\Lambda\Lambda}$ values about 3.6 MeV, since the $\Lambda\Lambda$ phases shifts predicted by both interactions increase upto about 40°. (The small difference between our results and the Hiyama's results, with respect to $V_{\Lambda\Lambda}$(Hiyama), arises, since the expressions of the $\Lambda\alpha$ exchange term are slightly different in these two calculations.) The improved quark model fss2 yields $\Delta B_{\Lambda\Lambda} = 1.41$ MeV. (If we use the $\Lambda\Lambda$ single-channel T-matrix, this number is reduced to $\Delta B_{\Lambda\Lambda} = 1.14$ MeV.) In Table 2, the result denoted by $V_{\Lambda\Lambda}$(SB) is by the 2-range Gaussian potential generated from the fss2 ${}^1S_0$ $\Lambda\Lambda$ phase shift in the full-channel calculation, using the supersymmetric inversion method. We think that the 0.5 MeV difference between our fss2 result and the $V_{\Lambda\Lambda}$(SB) result is probably because we neglected the full coupled-channel effect of the $\Lambda\Lambda\alpha$ channel to the $\Xi N\alpha$ and $\Sigma\Sigma\alpha$ channels. We should also keep in mind that in all of these 3-cluster calculations the Brueckner rearrangement effect [8] of the α-cluster with the magnitude of about -1 MeV (repulsive) is very important. Taking all of these effects into consideration, we can conclude that the present results by fss2 are in good agreement with the recent experimental value $\Delta B^{\mathrm{exp}}_{\Lambda\Lambda} = 1.01 \pm 0.20$ MeV [10] deduced from the Nagara event.

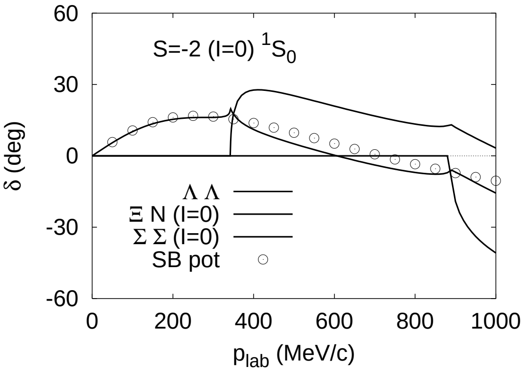

Figure 4. $\Lambda\Lambda$ ${}^1S_0$ phase shifts by fss2 (solid curve) and by SB potential (Circles).

6. SUMMARY

The 3-cluster Faddeev formalism using the 2-cluster RGM kernel opens a way to solve few-baryon systems interacting by the quark-model baryon-baryon interaction without spoiling the essential features of the RGM kernel; i.e., the non-locality, the energy depen-

Table 2
Comparison of $\Delta B_{\Lambda\Lambda}$ values in MeV, predicted by various $\Lambda\Lambda$ interactions and $V_{\Lambda N}$ potentials. FSS and fss2 use the $\Lambda\Lambda$ RGM T-matrix in the free space. $\varepsilon_{2\Lambda}$ is the 2Λ expectation value determined self-consistently. For $V_{\Lambda\Lambda}$(Hiyama) and $V_{\Lambda\Lambda}$(SB), see the text. $V_{\Lambda\Lambda}$(SB) uses only the S-wave, but others are the converged results with enough partial waves.

	$V_{\Lambda\Lambda}$(Hiyama)			FSS		fss2		$V_{\Lambda\Lambda}$(SB)
$V_{\Lambda N}$	ours	Hiyama	diff	E	$\varepsilon_{2\Lambda}$	E	$\varepsilon_{2\Lambda}$	(S-wave)
SB	3.618	–	–	3.650	5.131	1.411	5.942	1.902
NS	3.548	3.59	0.04	3.623	5.159	1.364	5.951	1.917
ND	3.181	3.10	−0.08	3.231	4.482	1.288	5.231	1.636
NF	3.208	3.22	0.01	3.301	4.626	1.270	5.410	1.719
JA	3.370	3.44	0.07	3.468	4.908	1.306	5.705	1.833
JB	3.486	3.56	0.07	3.592	5.150	1.325	5.956	1.924

dence and the existence of the pairwise Pauli-forbidden state. It can also be used for the 3-cluster systems involving α-clusters, like the ${}^{9}_{\Lambda}$Be system. A nice point of this formalism is that the underlying NN and YN interactions are more direcly related to the structure of the hypernuclei than the models assuming simple 2-cluster potentials. In particular, we have found that the most recent quark-model interaction, the model fss2, yields a realistic description of many systems including the 3-nucleon bound state, ${}^{9}_{\Lambda}$Be and ${}^{6}_{\Lambda\Lambda}$He.

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