1. Charmed baryons

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[状態分類]

Table 1: Spin-flavor $SU_6 \supset SU_2 \times SU_3$

[3]	$\frac{1}{2}(11)$, $\frac{3}{2}(30)$	56
[21]	$\frac{1}{2}(00)$, $\frac{1}{2}(30)$, $\frac{1}{2}(11)$, $\frac{3}{2}(11)$	70×2
[111]	$\frac{1}{2}(11)$, $\frac{3}{2}(00)$	20
dim.	$6 \times 6 \times 6 \;=$	216

Table 2: Spin-flavor $SU_8 \supset SU_2 \times SU_4$

[3]	$\frac{1}{2}(110)$, $\frac{3}{2}(300)$	120
[21]	$\frac{1}{2}(001)$, $\frac{1}{2}(300)$, $\frac{1}{2}(110)$, $\frac{3}{2}(110)$	168×2
[111]	$\frac{1}{2}(110)$, $\frac{3}{2}(001)$	56
dim.	$8 \times 8 \times 8 \;=$	512

Table 3: flavor $SU_4 \supset SU_3 \times U_1$

(300)	(30)	$(20)c$	$(10)cc$	$(00)ccc$
20 dim.	B_{10}	Σ^*-like	Ξ^*-like	Ω-like
(110)	(11)	$(01)c$	$(20)c$	$(10)cc$
20 dim.	B_8	Λ-like	Σ-like	Ξ-like
(001)	(00)	$(01)c$		
4 dim.	$\Lambda(00)$	$\Lambda(00)$-like		

[Wave functions and the Hamiltonian] (s-wave)

$$\begin{aligned} \Phi_{S(\lambda\mu)a}(3q) &= \phi^{\rm space}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)\, W^{[f]}_{S(\lambda\mu)a}(1,2,3)\, C_{(00)}(1,2,3)\ , \\ H &= \sum_{i=1}^{3} \left(m_i + \frac{{\boldsymbol{p}_i}^2}{2m_i} \right) - T_G + \sum_{i<j}^{3} v_{ij}\ , \end{aligned}$$

$$
\begin{aligned}
v_{ij} &= -a_c\, r^2\,(\lambda_i\,\lambda_j) + \alpha_S\,\hbar c\,\left\{\frac{1}{r} - \frac{\pi}{2}\left(\frac{\hbar}{c}\right)^2\left(\frac{1}{{m_i}^2}+\frac{1}{{m_j}^2}\right)\,\delta(\boldsymbol{r})\right.\\
&\quad -\frac{1}{2m_im_j}\left(\frac{\hbar}{c}\right)^2\,\frac{1}{r}\,\left[\boldsymbol{p}_i\cdot\boldsymbol{p}_j+\boldsymbol{r}(\boldsymbol{r}\cdot\boldsymbol{p}_i)\,\boldsymbol{p}_j\right]\\
&\quad \left.-\frac{1}{4m_im_j}\left(\frac{\hbar}{c}\right)^2\,\frac{8\pi}{3}\,(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\;\delta(\boldsymbol{r})\right\}\,\frac{1}{4}(\lambda_i\,\lambda_j)\;,\\
M_{S(\lambda\mu)a}(3q) &= \langle\Phi_{S(\lambda\mu)a}(3q)|H|\Phi_{S(\lambda\mu)a}(3q)\rangle = 24\,a_c\,b^2 - \sqrt{\frac{2}{\pi}}\alpha_S\left(\frac{\hbar c}{b}\right)\,2\\
&\quad +I_1+\frac{1}{2}\left(\frac{\hbar c}{b}\right)^2\,I_2+\sqrt{\frac{2}{\pi}}\alpha_S\left(\frac{\hbar c}{b}\right)^3\,\frac{2}{9}\,I_3\;. \qquad (1)
\end{aligned}
$$

where, for $(\lambda\mu)=(11)$,

$$
I_1=\left\{\begin{array}{lll} 2m_u+m_d & \text{for} & p\\ m_u+m_d+m_s & \text{for} & \Lambda,\ \Sigma^0\end{array}\right.\quad,\quad I_2=\left\{\begin{array}{lll} \frac{2}{m_u}+\frac{1}{m_d} & \text{for} & p\\ \frac{1}{m_u}+\frac{1}{m_d}+\frac{1}{m_s} & \text{for} & \Lambda,\ \Sigma^0\end{array}\right.
$$

$$
I_3=\left\{\begin{array}{lll} \frac{1}{{m_u}^2}+\frac{3}{4}\frac{1}{{m_d}^2}-\frac{4}{m_u\,m_d} & \text{for} & p\\ \frac{3}{4}\left(\frac{1}{{m_u}^2}+\frac{1}{{m_d}^2}+\frac{1}{{m_s}^2}\right)-\frac{5}{2}\frac{1}{m_u\,m_d}-\left(\frac{1}{m_u}+\frac{1}{m_d}\right)\frac{1}{m_s} & \text{for} & \Lambda\\ \frac{3}{4}\left(\frac{1}{{m_u}^2}+\frac{1}{{m_d}^2}+\frac{1}{{m_s}^2}\right)-\frac{1}{2}\frac{1}{m_u\,m_d}-2\left(\frac{1}{m_u}+\frac{1}{m_d}\right)\frac{1}{m_s} & \text{for} & \Sigma^0\end{array}\right. \qquad (2)
$$

The other factors are obtained by

$$
\begin{array}{rcl} n & : & u\leftrightarrow d \quad \text{in} \quad p\\ \Sigma^+ & : & d\to s \quad \text{in} \quad p\ ,\quad \Sigma^- \ : \ u\to s \quad \text{in} \quad n\\ \Xi^0 & : & d\to s \quad \text{in} \quad n\ ,\quad \Xi^- \ : \ u\to s \quad \text{in} \quad p\end{array} \qquad (3)
$$

When $(\lambda\mu)=(11)$, we use

$$
I_1=\left\{\begin{array}{lll} 3m_u & \text{for} & \Delta^{++}\\ 2m_u+m_d & \text{for} & \Delta^+\\ m_u+m_d+m_s & \text{for} & \Sigma^0\end{array}\right.\quad,\quad I_2=\left\{\begin{array}{lll} \frac{3}{m_u} & \text{for} & \Delta^{++}\\ \frac{2}{m_u}+\frac{1}{m_d} & \text{for} & \Delta^+\\ \frac{1}{m_u}+\frac{1}{m_d}+\frac{1}{m_s} & \text{for} & \Sigma^0\end{array}\right.
$$

$$
I_3=\left\{\begin{array}{lll} \frac{3}{4}\frac{1}{{m_u}^2} & \text{for} & \Delta^{++}\\ \frac{1}{{m_u}^2}+\frac{3}{4}\frac{1}{{m_d}^2}-\frac{1}{m_u\,m_d} & \text{for} & \Delta^+\\ \frac{3}{4}\left(\frac{1}{{m_u}^2}+\frac{1}{{m_d}^2}+\frac{1}{{m_s}^2}\right)-\frac{1}{2}\left(\frac{1}{m_u\,m_d}+\frac{1}{m_u\,m_s}+\frac{1}{m_d\,m_s}\right) & \text{for} & \Sigma^0\end{array}\right. \qquad (4)
$$

The other factors are obtained by

$$
\Delta^0 \ : \ u\leftrightarrow d \quad \text{in} \quad \Delta^+\ ,\quad \Delta^- \ : \ u\to d \quad \text{in} \quad \Delta^{++}
$$

Table 4: Quark-model parameters

$b = 0.655$ fm	
$m_{ud} = 313$ MeV	$x = 0.9624$
$\lambda_s = 1.65$	$m_s c^2 = 516$ MeV
$\lambda_c = 5.375$	$m_c c^2 = 1682$ MeV
$\sqrt{\frac{2}{\pi}}\alpha_S^c x^3 m_{ud} c^2 = 470$ MeV	$\sqrt{\frac{2}{\pi}}\alpha_S x^3 m_{ud} c^2 = 440$ MeV
$\alpha_S^c = 2.111$	$\alpha_S = 1.977$

$$\begin{aligned} \Sigma^+ &: d \to s \quad \text{in} \quad \Delta^+ \quad , \quad \Sigma^- : u \to s \quad \text{in} \quad \Delta^0 \\ \Xi^0 &: d \to s \quad \text{in} \quad \Delta^0 \quad , \quad \Xi^- : u \to s \quad \text{in} \quad \Delta^+ \\ \Omega^- &: u \to s \quad \text{in} \quad \Delta^{++} \end{aligned} \tag{5}$$

[パラメータの決定] $\quad x = \frac{\hbar}{m_{ud} bc}$

1. $\Delta E_{N\Delta} = \sqrt{\frac{2}{\pi}}\alpha_S x^3 m_{ud} c^2 \, \frac{2}{3} = \Delta - N = 1232 - 939 = 293$
   $\to \sqrt{\frac{2}{\pi}}\alpha_S x^3 m_{ud} c^2 = 440$ MeV

2. $\Delta E_{\Lambda\Sigma} = \sqrt{\frac{2}{\pi}}\alpha_S x^3 m_{ud} c^2 \left(1 - \frac{1}{\lambda}\right) \frac{4}{9} = \Sigma - \Lambda = 1193 - 1116 = 77$
   $\to \lambda = 1.65$

3. $\Delta E_{N\Lambda} = m_{ud} c^2 \left(1 - \frac{1}{\lambda}\right) \left[\lambda - \frac{x^2}{2} + \sqrt{\frac{2}{\pi}}\alpha_S x^3 \, \frac{1}{6}\left(\frac{5}{3} - \frac{1}{\lambda}\right)\right] = \Lambda - N = 1116 - 939 = 177$
   $\Delta E_{\Delta\Sigma^*} = m_{ud} c^2 \left(1 - \frac{1}{\lambda}\right) \left[\lambda - \frac{x^2}{2} + \sqrt{\frac{2}{\pi}}\alpha_S x^3 \, \frac{1}{6}\left(\frac{1}{3} - \frac{1}{\lambda}\right)\right] = \Sigma^* - \Delta = 1386 - 1232 = 154$
   $m_{ud} c^2 \left(1 - \frac{1}{\lambda}\right) \left(\lambda - \frac{x^2}{2}\right) = 146 \quad \cdots \Delta E_{N\Lambda}$ をあわす (B_{10} は少し下りすぎ)
   $\to b$ と m_{ud} との関係を与える。
   Set I: $b = 0.655$ fm, $m_{ud} c^2 = 313$ MeV, $x = 0.9624$
   Set II: $b = 0.6$ fm, $m_{ud} c^2 = 326$ MeV, $x = 1.0087$
   どちらでやっても結果はほとんど変らないので以下 Set I で行く。

4. $\Delta E_{N\Lambda_c}$, $\Delta E_{\Lambda_c\Sigma_c}$ で λ_c と α_S^c を決定する。

結果を Table 5, 6 に示す。Table 7 はこれら以外の見つかっている charmed baryon である。

Table 5: Calculated energies of uds baryons (S-wave). The undelined numbers are fitted.

B_8	$M^{\rm cal}$ (MeV)	cal-exp	B_{10}	$M^{\rm cal}$ (MeV)	cal-exp
$N(939)$	<u>939</u>	–	$\Delta(1232)$	<u>1232</u>	–
$\Lambda(1116)$	<u>1116</u>	–	$\Sigma^*(1386)$	1370	−16
$\Sigma(1193)$	<u>1193</u>	–	$\Sigma^*(1535)$	1500	−33
$\Xi(1318)$	1324	6	$\Omega(1672)$	1624	−48

2. *P*-wave and positive-parity baryons

Particle Data Group, Phys. Lett. B592, 1 (2004), (http://pdg.lgl.gov), Summary table

1. Y. Fujiwara, RGM in Quark-Hadron Physics — The Missing LS Force Problem in P-Wave Baryons — , in *Proceedings of Microscopic Cluster Models of Light Muclei and Related Topics*, Soryushiron Kenkyu (Kyoto) **87** (1993), B147 - B168.

2. Y. Fujiwara, Baryon-Meson Couplings in the $q\bar{q}$ Pair-Creation Quark Model. I — Coupling Constants of S-Wave Baryons — , Prog. Theor. Phys. **88** (1992), 933 - 965.

3. Y. Fujiwara, Baryon-Meson Couplings in the $q\bar{q}$ Pair-Creation Quark Model. II — Mesonic Decay Widths of P-Wave Baryons — , Prog. Theor. Phys. **89** (1993), 455 - 492.

4. Y. Fujiwara, Baryon-Meson Couplings in the $q\bar{q}$ Pair-Creation Quark Model. III — Energy Shifts of P-Wave Baryons — , Prog. Theor. Phys. **90** (1993), 105 - 151.

Table 8 に Ref. [4] によって assign された usd sector の P-wave baryon の分類をしめす。これにより、2 GeV 以下のほとんど全ての negative parity states は quark model で過不足なく説明できる。一方、Table 9 は Roper resonance $N(1440)$ を始めとする (B_8, B_{10} 以外の) positive parity resonance でこれらについては、今のところ構成子クォーク模型によるまともな説明はない。

Table 6: Calculated energies of charmed baryons (S-wave). The undelined numbers are fitted.

J^π	B	$M^{\rm cal}$ (MeV)	$M^{\rm cal} - M^{\rm exp}$ (MeV)
$\frac{1}{2}^+$	$\Lambda_c(2285)$	$\underline{2285}$	–
Λ-like	$\Xi_c(2469)$	2492	23
$\frac{1}{2}^+$	$\Sigma_c(2455)$	$\underline{2455}$	–
Σ-like	$\Xi_c'(2577)$	2587	10
	$\Omega_c^0(2698)$	2711	13
$\frac{1}{2}^+$	$\Xi_{cc}(3519)$	3681	162
Ξ-like	Ω_{cc}^+	3808	
$\frac{3}{2}^+$	$\Sigma_c(2520)$	2492	-28
Σ-like	$\Xi_c(2645)$	2613	-32
	Ω_c^0	2725	
$\frac{3}{2}^+$	Ξ_{cc}	3717	
Ξ-like	Ω_{cc}^+	3821	
$\frac{3}{2}^+$	Ω_{ccc}^{++}	4908	

Table 7: Other charmed baryons. The upper two Λ do not exist in the summary table. The nagative parity baryons correspond to $\Lambda(1405)$ and $\Lambda(1520)$.

$\Lambda_c(2765)$	$\Lambda_c(2880)$
Negative parity charm baryons	
$\Lambda_c(2593)$ 0 $\frac{1}{2}^-$	$\Lambda_c(2625)$ 0 $\frac{3}{2}^-$
$\Xi_c(2790)$ $\frac{1}{2}$ $\frac{1}{2}^-$	$\Xi_c(2815)$ $\frac{1}{2}$ $\frac{3}{2}^-$

Table 8: P-wave uds baryons

$S(\lambda\mu)$	$B\ L_{2I+1,2S+1}$ or $L_{I,2S+1}$			
$\frac{1}{2}(00)$	$\Lambda(1405)\ S_{01}$			
	$\Lambda(1520)\ D_{03}$			
$\frac{1}{2}(30)$	$\Delta(1620)\ S_{31}$	$\Sigma(1750)\ S_{11}$	$\Xi(1950)$?	Ω ?
	$\Delta(1700)\ D_{33}$	$\Sigma(1940)\ D_{13}$	$\Xi(2030)$?	($\Omega(2250)$?)
$\frac{1}{2}(11)$	$N(1535)\ S_{11}$	$\Lambda(1670)\ S_{01}$	Σ ?	Ξ ?
	$N(1520)\ D_{13}$	$\Lambda(1690)\ D_{03}$	$\Sigma(1670)\ D_{13}$	$\Xi(1820)$?
$\frac{3}{2}(11)$	$N(1650)\ S_{11}$	$\Lambda(1800)\ S_{01}$	Σ ?	Ξ ?
	$N(1675)\ D_{15}$	$\Lambda(1830)\ D_{05}$	$\Sigma(1775)\ D_{15}$	Ξ ?
	$N(1700)\ D_{13}$	Λ ?	Σ ?	Ξ ?
extra	$\Xi(1690)\ \frac{1}{2}?^{?}$			
	$N(2190)\ G_{17}$	$\Lambda(2100)\ G_{07}$		
	$N(2250)\ G_{19}$			
	$N(2600)\ I_{1,11}$			

Table 9: Positive-parity baryons

$B\ L_{2I+1,2S+1}$ or $L_{I,2S+1}$			
$N(1440)\ P_{11}$	$\Delta(1600)\ P_{33}$	$\Lambda(1600)\ P_{01}$	$\Sigma(1660)\ P_{11}$
$N(1680)\ F_{15}$	$\Delta(1905)\ F_{35}$	$\Lambda(1810)\ P_{01}$	$\Sigma(1915)\ F_{15}$
$N(1710)\ P_{11}$	$\Delta(1910)\ P_{31}$	$\Lambda(1820)\ F_{05}$	$\Sigma(2030)\ F_{17}$
$N(1720)\ P_{13}$	$\Delta(1920)\ P_{33}$	$\Lambda(1890)\ P_{03}$	$\Sigma(2250)\ 1?^{?}$
$N(2220)\ H_{19}$	$\Delta(1950)\ F_{37}$	$\Lambda(2110)\ F_{05}$	
	$\Delta(2420)\ H_{3,11}$	$\Lambda(2350)\ H_{09}$	
$\theta(1540)^{+}$			