Faddeev Calculation of the Hypertriton in the Quark-Model NN and YN Interactions *

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The recent quark-model nucleon-nucleon (NN) and hyperon-nucleon (YN) interactions by the Kyoto-Niigata group is applied to the hypertriton calculation in a new three-cluster Faddeev formalism using the two-cluster resonating-group method (RGM) kernels. The most recent model, fss2, gives a reasonable result similar to the Nijmegen soft-core model NSC89.

1. INTRODUCTION

The QCD-inspired spin-flavor SU_6 quark model for the baryon-baryon interaction, proposed by the Kyoto-Niigata group, is a unified model for the complete baryon octet ($B_8 = N$, Λ, Σ and Ξ), which has achieved very accurate descriptions of the NN and YN interactions. [1–3] In particular, the NN interaction of the most recent model fss2 [2] is accurate enough to compare with the modern realistic meson-exchange models. These quark-model interactions can be used for realistic calculations of few-baryon and few-cluster systems, once an appropriate three-body equation is formulated for the pairwise interactions described by the RGM kernel. The desired 3-cluster equation should be able to deal with the non-locality and the energy-dependence intrinsically involved in the quark-exchange RGM kernel. Furthermore, the quark-model description of the YN and hyperon hyperon (YY) interactions in the full coupled-channel formalism sometimes involves a Pauli forbidden state at the quark level, which excludes the most compact $(0s)^6$ spatial configuration, resulting in the strongly repulsive nature of the interactions in some particular channels. We have recently formulated a new 3-cluster equation which uses two-cluster RGM kernels explicitly. [4] This equation exactly eliminates 3-cluster redundant components by the orthogonality of the total wave function to the pairwise 2-cluster Pauli-forbidden states. The explicit energy dependence inherent in the exchange RGM kernel is self-consistently determined. This equation is entirely equivalent to the Faddeev

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equation which uses a singularity-free T-matrix (which we call the RGM T-matrix) constructed from the 2-cluster RGM kernel. We first applied this formalism to a 3 di-neutron system and the 3α system, and obtained complete agreement between the Faddeev calculations and variational calculations using the translationally invariant harmonic-oscillator (h.o.) basis. [4,5] This formalism was also applied to the Faddeev calculation of the $3N$ bound state [6], employing the off-shell T-matrices which are derived from the non-local and energy-dependent RGM kernels for our quark-model NN interactions of fss2 and FSS. The model fss2 yields the triton binding energy $B_t = 8.519$ MeV in the 50 channel calculation, when the np interaction is employed for any NN pairs in the isospin basis. [7] If we take into account the charge dependence of the NN interaction, this triton binding energy is the closest to the experimental value, ${B_t}^{\rm exp} = 8.48$ MeV, among many Faddeev calculations employing modern realistic NN interactions. The charge rms radii for ^{3}H and ^{3}He are also correctly reproduced. A characteristic description of the short-range correlations in the quark model is essential to reproduce the large binding energy and the correct size of the three-nucleon bound state without reducing the D-state probability of the deuteron.

Here we apply our quark-model NN and YN interactions to the hypertriton with the small separation energy of the Λ-particle, ${B_\Lambda}^{\rm exp} = 130 \pm 50$ keV. Since the Λ-particle is far apart from the two-nucleon subsystem, the on-shell properies of the ΛN and ΣN interactions are supposed to be well reflected in this system. In particular, this system is very useful to know the relative strength of $^1S_0$ and $^3S_1$ attractions of the ΛN interaction, since the $^1S_0$ component of the ΛN interaction is more effective in the hypertriton and the available low-energy Λp total cross sections cannot discriminate many possible combinations of the $^1S_0$ and $^3S_1$ interactions. In fact, Ref. [8,9] shows that many meson-theoretical interactions fail to bind the hypertriton, except for the Nijmegen soft-core potentials NSC89 [10], NSC97f and NSC97e [11]. It is also pointed out in [8,9] that a small admixture of the ΣNN components less than 1% is very important for this binding.

2. FORMUATION

The total wave function of the hypertriton system is expressed as the antisymmetric superposition of two independent Faddeev components ψ and φ: $\Psi = \psi + (1 - P_{12})\varphi$. The two nucleons are numbered 1 and 2, the Λ or Σ is numbered 3. All of these wave functions are two dimensional vectors composed of the ΛNN and ΣNN components. [8] The coupled-channel Faddeev equation reads

$$\psi = G_0 t (1 - P_{12}) \varphi \quad , \qquad \varphi = G_0 \widetilde{\tau} \left[\psi - P_{12} \varphi \right] \qquad . \tag{1}$$

Here, t is the NN T-matrix in the 3-body space, and $\widetilde{\tau}$ is the redundancy-free ΛN-ΣN $\widetilde{T}$-matrix in the coupled-channel formalism. These T-matrices are generated from the NN and YN RGM kernels $V^{\rm RGM}_{NN}(\varepsilon_{NN})$ and $V^{\rm RGM}_{YN}(\varepsilon_{YN})$, respectively, according to the prescription given in [4]. The energy dependence involved in these kernels is determined self-consistently by calculating the matrix elements of the 2-cluster Hamiltonian as

$$\varepsilon_{NN} = \langle \Psi | h_{NN} + V^{\rm RGM}_{NN}(\varepsilon_{NN}) | \Psi \rangle \quad , \qquad \varepsilon_{YN} = \langle \Psi | h_{YN} + V^{\rm RGM}_{YN}(\varepsilon_{YN}) | \Psi \rangle \quad . \tag{2}$$

Table 1
Results of ${}^3_\Lambda$H Faddeev calculations by fss2 and FSS. The calculated deuteron binding energy is $\varepsilon_d = 2.2247$ MeV for fss2 and 2.2561 MeV for FSS ($\varepsilon_d^{\rm exp} = 2.2246$ MeV). The heading E is the ${}^3_\Lambda$H energy measured from the $2N + \Lambda$ threshold; B_Λ is the Λ separation energy; ε_{NN} ($\varepsilon_{\Lambda N}$) is the NN (ΛN) expectation value determined self-consistently; and P_Σ is the ΣNN probability in percent. The norm of redundant components, $N^2_{\rm red} = \langle f|f\rangle$ with $f = \langle (11)_s|\Psi\rangle$, is less than 10^{-9}.

model	channels	E (MeV)	B_Λ (keV)	ε_{NN} (MeV)	$\varepsilon_{\Lambda N}$ (MeV)	P_Σ (%)
	15 ch (SD)	−2.423	198	−1.762	5.729	0.652
fss2	54 ch ($J \leq 2$)	−2.498	273	−1.673	5.974	0.777
	102 ch ($J \leq 4$)	−2.513	288	−1.658	6.022	0.804
	150 ch ($J \leq 6$)	−2.514	289	−1.657	6.024	0.805
	15 ch (SD)	−2.967	710	−1.433	6.171	1.200
FSS	54 ch ($J \leq 2$)	−3.121	865	−1.323	6.467	1.348
	102 ch ($J \leq 4$)	−3.134	877	−1.317	6.488	1.360
	150 ch ($J \leq 6$)	−3.134	878	−1.317	6.489	1.361

For the present calculation, exact treatment of the ΛN-ΣN coupling and the resulting Pauli forbidden state with the SU_3 $(11)_s$ symmetry is very important to obtain precise results. For this reason, we keep the free-space relationship $\varepsilon_{\Lambda N} - \varepsilon_{\Sigma N} = \Delta E_{\Sigma\Lambda}$ with $\Delta E_{\Sigma\Lambda} = M_\Sigma c^2 - M_\Lambda c^2$, even in the 2-body equations and $\widetilde{\tau}$-matrix in the 3-body space. Then we find $\varepsilon_{\Lambda N} = \varepsilon_{YN} + \Delta E_{\Sigma\Lambda} P_\Sigma$, where $P_\Sigma = \langle \Psi_{\Sigma NN}|\Psi_{\Sigma NN}\rangle$ is the probability of the ΣNN component admixed in the hypertriton wave function.

3. RESULT

Table 1 shows the results of the Faddeev calculations using fss2 and our previous model FSS. In the 15 channel calculation including the S and D waves of the NN and YN interactions, we have already achieved B_Λ $(= -\varepsilon_d - E({}^3_\Lambda{\rm H})) \sim 200$ keV for fss2. The convergence with the extension to the higher partial waves is very rapid, and the total angular-momentum $J \leq 4$ is good enough for the 1 keV accuracy. The momentum descritization points are n_1-n_2-n_3=10-10-5 with q_1-q_2-q_3=1-3-6 fm^{-1} in the notation given in [6]. For the converged B_Λ values with 150 channel ΛNN and ΣNN configurations, we find $B_\Lambda = 289$ keV and the ΣNN component $P_{\Sigma NN} = 0.80\,\%$ for the fss2 prediction, and $B_\Lambda = 878$ keV and $P_{\Sigma NN} = 1.36\,\%$ for FSS.

Table 2 shows the correlation between the Λ separation energy B_Λ and the ${}^1S_0$ and ${}^3S_1$ effective range parameters of various ΛN interactions. Although all of these ΛN interactions reproduce the low-energy ΛN total cross section data below $p_\Lambda = 300$ MeV/c within the experimental error bars, our quark-model interactions are more attractive than the Nijmegen soft-core potential NSC89 [10]. The model FSS gives a large overbinding since the ${}^1S_0$ ΛN interaction is strongly attractive. The phase-shift difference of the ${}^1S_0$ and ${}^3S_1$ states at $p_\Lambda \sim 200$ MeV/c is $\delta({}^1S_0) - \delta({}^3S_1) \sim 30^\circ$ for FSS, while $\delta({}^1S_0) - \delta({}^3S_1) \sim$

Table 2
The $^1S_0$ and $^3S_1$ effective range parameters of various ΛN interactions and the Λ separation energy B_Λ of the hypertriton. The B_Λ value of NSC89 is taken from Ref. [9].

model	a_s (fm)	r_s (fm)	a_t (fm)	r_t (fm)	B_Λ (keV)
FSS [1]	−5.39	2.26	−1.02	4.20	878
fss2 [2,3]	−2.58	2.83	−1.60	3.00	289
NSC89 [10]	−2.78	2.88	−1.41	3.11	143

10° for fss2. Since the present fss2 result is still slightly overbound, this difference should be much smaller in order to reproduce the correct experimental value ${B_\Lambda}^{\rm exp} = 130 \pm 50$ keV. From the two results given by fss2 and FSS, we can extrapolate the desired difference to be $3^\circ \sim 7^\circ$, which is consistent with the result in Ref. [12] using simulated interactions of the Nijmegen models.

Table 1 also shows that the expectation value of the NN Hamiltonian, ε_{NN}, is rather close to the deuteron energy $-\varepsilon_d$, especially in fss2. This resemblance is even marked if we decompose these energies to the kinetic-energy and potential-energy contributions:

$$\varepsilon_{NN} = 19.376 - 21.032 = -1.657 \text{ MeV} \quad , \qquad -\varepsilon_d = 17.495 - 19.720 = -2.225 \text{ MeV} \quad . \tag{3}$$

Namely, the NN subsystem shrinks by the effect of the outer Λ-particle, resulting in the increase of the kinetic energy by 2.10 MeV. This situation is very similar to the results of the NSC89 potential. In this case, we find [8]

$$\varepsilon_{NN} = 20.48 - 22.25 = -1.77 \text{ MeV} \quad , \qquad -\varepsilon_d = 19.304 - 21.528 = -2.224 \text{ MeV} \quad . \tag{4}$$

In fact, the similarity between our fss2 and the NSC89 results already appears in the decomposition of the total hypertriton energies to the kinetic-energy and potential-energy contributions:

$$\begin{aligned} E(\text{FSS}) &= 27.372 - 30.506 = -3.134 \text{ MeV} \quad , \\ E(\text{fss2}) &= 23.860 - 26.374 = -2.514 \text{ MeV} \quad , \\ E(\text{NSC89}) &= 23.45 - 25.79 = -2.34 \text{ MeV} \quad . \end{aligned} \tag{5}$$

However, the effect of the higher partial waves than S and D waves is fairly large in our quark-model potentials fss2 and FSS.

4. SUMMARY

In this study, we have carried out the Faddeev calculations, using the most recent quark-model NN and YN interactions, FSS [1] and fss2 [2,3]. These are realistic interactions which describe all the available NN and YN data, incorporating the effective meson-exchange potentials at the quark level. Since these quark-model baryon-baryon interactions are formulated in the RGM framework, they are non-local, energy-dependent, and they sometimes involve the Pauli-forbidden component at the quark level. The hypertriton is an appropriate place to investigate the roles of the compact SU_3 $(11)_s$ state, which is completely Pauli-forbidden in the ΛN-ΣN coupling with the isospin $I = 1/2$. In order to take into account this off-shell effect of the quark-model interaction, we have formulated a new type of the Faddeev equation which explicitly uses the 2-cluster RGM kernels. [4,5] The energy-dependence of the RGM kernel is self-consistently determined, by calculating the expectation values of the 2-cluster Hamiltonian with respect to the obtained solutions of the Faddeev equation. The present study is the second application of this formalism to the few-baryon systems interacting via the quark-model baryon-baryon interactions, followed by our previous one to the triton system [6,7].

We have found that the most recent model, fss2, gives a reasonable result for the hypertriton properties, which is very similar to the results of the Nijmegen soft-core models NSC89 [10] and NSC97f [11]. The Λ separation energy given by fss2 is $B_\Lambda = 289$ keV, which is a little too large compared with the experimental value ${B_\Lambda}^{\rm exp} = 130{\pm}50$ keV. Unlike the NSC89 result, the effects of higher partial waves up to the G wave are rather important in the quark-model NN and YN interactions. If we use the dominant S-wave character of the ΛN interaction in the hypertriton system, the $^1S_0$ ΛN interaction of the model fss2 is slightly too attractive. It is a future problem to clarify if such an improvement as to reduce the $^1S_0$ attraction is a favorable feature to reproduce the 0^+ and 1^+ states of ${}^4_\Lambda$H and ${}^4_\Lambda$He systems. The fairly large charge symmetry breaking in these systems is also an important aspect to understand the hyperon-nucleon interactions in detail.

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