A Consistent 3α and $2\alpha\Lambda$ Faddeev Calculation using the 2α RGM Kernel*

Y. Fujiwara[a], K. Miyagawa[b], M. Kohno[c], Y. Suzuki[d], D. Baye[e] and J.-M. Sparenberg[f]

[a]Department of Physics, Kyoto University, Kyoto 606-8502, Japan

[b]Department of Applied Physics, Okayama Science University, Okayama 700-0005, Japan

[c]Physics Division, Kyushu Dental College, Kitakyushu 803-8580, Japan

[d]Department of Physics, Niigata University, Niigata 950-2181, Japan

[e]Physique Nucléaire Théorique et Physique Mathématique, CP 229, Université Libre de Bruxelles, B-1050 Brussels, Belgium

[f]TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3

A Consistent description of the 3α and $2\alpha\Lambda$ systems is attempted in the three-cluster Faddeev Formalism, which explicitly employs the 2α resonating-group-method (RGM) kernel reproducing the empirical $\alpha\alpha$ phase shifts. The 3-range Minnesota (MN) force for the effective NN interaction gives a larger 3α binding energy than the 2-range Volkov No.1 and No.2 forces. The $2\alpha\Lambda$ ground-state and 2^+ excited-state energies are reasonably reproduced for the 3-range MN force, if an appropriate ΛN force is used for the $\Lambda\alpha$ folding potential. Here, the effective ΛN force is spin-singlet and triplet two-range Gaussian central forces, constructed from the phase-shift behavior of the quark-model hyperon-nucleon (YN) interaction, fss2, by using an inversion method based on supersymmetric quantum mechanics.

1. INTRODUCTION

In spite of much effort to incorporate microscopic features of the $\alpha\alpha$ interaction, a consistent description of 3α and $2\alpha\Lambda$ systems has not yet been obtained so far in the Faddeev formalism. The 3α system suffers from a large overbinding for the ground state and sometimes involves spurious states because of the incomplete treatment of the Pauli-forbidden states in the 3α model space. [1–3] We have recently formulated a new 3-cluster equation which uses 2-cluster RGM kernels explicitly. [4] This equation exactly eliminates 3-cluster redundant components by requiring the orthogonality of the total wave function to the pairwise 2-cluster Pauli-forbidden states. The explicit energy dependence inherent in the exchange RGM kernel is self-consistently treated. This equation is entirely equivalent to the Faddeev equation which uses a singularity-free T-matrix (which we call the RGM T-matrix) constructed from the 2-cluster RGM kernel. We first applied this formalism to

*This work was supported by Grants-in-Aid for Scientific Research (C) from the Japan Society for the Promotion of Science (JSPS), Nos. 15540270, 15540284, 15540292.

3-dineutron and 3α systems, and obtained complete agreement between the Faddeev calculations and variational calculations which use the translationally invariant harmonic-oscillator (h.o.) basis. [4,5]

2. 3α FADDEEV CALCULATION

The input for the 3α calculation is an effective NN force for the 2α RGM kernel, for which we examine three different types of forces, the 3-range MN force [6] and the 2-range Volkov No.1 and No.2 forces. These are selected on purpose to compare the present 3-cluster formalism with the fully microscopic RGM [7,8] and GCM [9] calculations. The exchange-mixture (or Majorana) parameters and the h.o. width parameters of the $(0s)^4$ α clusters are given in Table 1. The 2α RGM calculations using these effective NN forces reasonably reproduce the empirical 2α phase shifts,

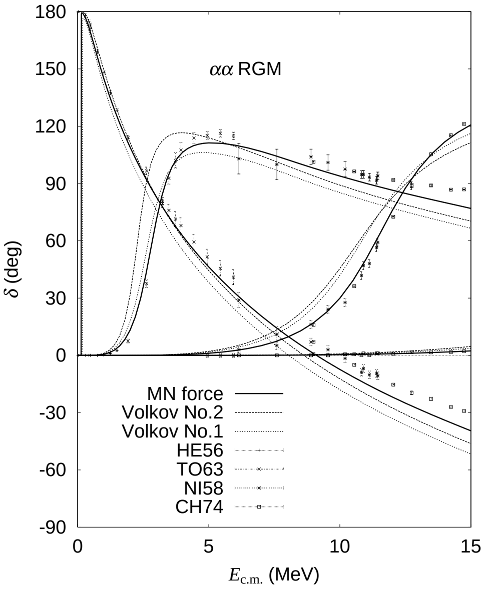

Figure 1. 2α phase shifts calculated by the 3-range Minnesota (MN) force (thick solid curves) and the Volkov forces (others).

as shown in Fig. 1. However, the best fit to the experiment is obtained by the 3-range MN force. For the Volkov No.2 force, the S-wave resonance appears as a bound state with the binding nergy $B_{2\alpha} = 245$ keV. Although the Volkov No.1 force reproduces the correct resonance energy of the S-wave, the overall fit to the 2α phase shifts is less perfect than the MN force. In the 3α Faddeev calculation, we use the cut-off Coulomb force $v^{\mathrm{C}}(r) = e^2/r \cdot \theta(R_C - r)$ with $R_C = 10$ fm for the 2α Coulomb kernel. For the 3α ground state, this approximation is good enough to obtain 1 - 2 keV accuracy.

Table 1 shows the 3α ground-state energies $E_{3\alpha}$, predicted in the present 3-cluster Faddeev formalism, compared with those obtained by fully microscopic calculations. We find that the present 3-cluster equation gives the 3α energies which are only 1.5 - 1.7 MeV higher than those of the fully microscopic 3α RGM or GCM calculations. This implies that the 3-cluster exchange effect, which is neglected in our 3-cluster formalism, but is present in the full microscopic 3-cluster RGM kernel, is attractive in nature and is not so strong as the repulsive 3-body force claimed necessary in the semi-microscopic 3α models [10,11]. This is mainly because the 3α equations used by these authors do not eliminate 3α Pauli-forbidden components accurately, unlike the one used in the present Faddeev formalism. In Table 1, we also find that the 3-range MN force gives a somewhat large overbinding of 2 - 4 MeV, if the 3α energy $E_{3\alpha}$ is measured from the 3α threshold. In this respect, it is interesting to note that the non-compact α clusters with $\nu = 0.257$ fm^{-2} give less binding in the framework of the orthogonality condition model (OCM). [12] If the h.o. constant parameter ν is small, the proper treatment of the 2α exchange kernel seems to be necessary in order to reproduce the large ground-state energy $E_{3\alpha}$.

Table 1
The 3α ground-state energies $E_{3\alpha}$, measured from the 3α threshold ($E^{\rm exp}_{3\alpha} = -7.275$ MeV), in the present model and those by fully microscopic calculations. The present model is the Faddeev calculation using the 2α RGM kernel, including the cut-off Coulomb force with $R_C = 10$ fm. The effective NN forces are Volkov No.1 (VN1) and No.2 (VN2) forces with the Majorana parameter m, and the 3-range Minnesota (MN) force with the exchange-mixture parameter u. The heading E_α implies the internal energy of the $(0s)^4$ α cluster with the h.o. width parameter ν. The heading $E_{\rm tot}$ is the total energy from the RGM or GCM calculations; $E^{\rm full}_{3\alpha} = E_{\rm tot} - 3E_\alpha$; and "diff." stands for the energy difference between our model and the fully microscopic calculation, $E_{3\alpha} - E^{\rm full}_{3\alpha}$. The unit of energies is in MeV.

force	m or u	ν (fm^{-2})	E_α	$E_{3\alpha}$	$E_{\rm tot}$	$E^{\rm full}_{3\alpha}$	diff.
VN1	0.575	0.2515	-27.0	-5.38	-87.9 [9]	-6.9	1.5
VN2	0.59	0.275	-27.3	-5.98	-89.4 [7]	-7.5	1.5
MN	0.946869	0.257	-23.9	-9.64	-83.0 [8]	-11.3	1.7

3. $2\alpha\Lambda$ FADDEEV CALCULATION

The formalism is now applied to the $2\alpha\Lambda$ Faddeev calculation for ${}^{9}_{\Lambda}$Be, by using the 2α RGM kernel with the 3-range MN force and $\Lambda\alpha$ folding potentials with various ΛN effective forces. The effective ΛN force, denoted by SB in Table 2, is constructed from the ${}^1S_0$ and ${}^3S_1$ phase shifts predicted by the quark-model YN interaction, fss2 [13], by using an inversion method based on supersymmetric quantum mechanics [14]. It consists of two simple 2-range Gaussian potentials which reproduce the low-energy behavior of the ΛN phase shifts predicted in the full coupled-channel calculations. Since any central

Table 2
The ground-state energy $E_{\rm gr}(0^+)$ and the 2^+ excitation energy $E_{\rm x}(2^+)$ in MeV, calculated by solving the Faddeev equation for the $2\alpha\Lambda$ system. The rightmost ΔE implies a simple estimate of the ℓs splitting for the $5/2^+$ and $3/2^+$ excited states, obtained by using the $\boldsymbol{P}$=0 Wigner transform of the fss2 and FSS ΛN LS kernels. The model fss2 involves an extra contribution from the σ-meson exchange, which is indicated by "$+\sigma$".

$V_{\Lambda N}$	$E_{\rm gr}(0^+)$ (MeV)			$E_{\rm x}(2^+)$	ΔE (keV)	
	ours	Hiyama	diff.	(MeV)	fss2	FSS
SB	-6.837	–	–	2.915	$103+\sigma$	164
NS	-6.742	-6.81	0.07	2.916	$97+\sigma$	154
ND	-7.483	-7.57	0.09	2.935	$106+\sigma$	169
NF	-6.906	-7.00	0.09	2.930	$86+\sigma$	138
JA	-6.677	-6.76	0.08	2.919	$86+\sigma$	138
JB	-6.474	-6.55	0.08	2.911	$85+\sigma$	136
Exp't	-6.62 ± 0.04			3.029(3)/3.060(3)	43 ± 5	

and single-channel effective ΛN forces lead to the well-known overbinding problem of ${}^{5}_{\Lambda}$He [15] by about 2 MeV (in the present case, it is 1.63 MeV), the attractive part of the ${}^{3}S_{1}$ ΛN potential is adjusted to reproduce the empirical Λ-separation energy $B^{\rm exp}_{\Lambda}({}^{5}_{\Lambda}{\rm He}) = 3.12 \pm 0.02$ MeV. The odd-state ΛN force is assumed to be zero. The partial waves up to $\lambda_{\rm Max} = \ell_{1 \rm Max} = 6$ are included both in the 2α and $\Lambda\alpha$ channels. Table 2 shows the ground-state and excitation energies of ${}^{9}_{\Lambda}$Be, predicted by the SB and the other various ΛN potentials used by Hiyama *et al.*[16] In the present calculation using only central forces, the SB potential with the pure Serber character can reproduce the ground-state and excitation energies within the accuracy of 100 - 200 keV. Table 2 also shows a simple estimate of the ℓs splitting for the $5/2^{+}$ and $3/2^{+}$ excited states, due to the spin-orbit interactions predicted by fss2 and FSS.

4. OTHER APPLICATIONS

The 3-cluster Faddeev formalism using the 2-cluster RGM kernel opens a way to solve few-baryon systems interacting via quark-model baryon-baryon interactions without spoiling some essential features of the RGM kernel; i.e., the non-locality, the energy dependence and the existence of the pairwise Pauli-forbidden state in some special channels. Here we have applied this formalism to the 3-cluster systems involving α-clusters; i.e., the 3α and $2\alpha\Lambda$ systems. A nice point of this formalism is that the underlying NN and YN interactions are more directly related to the structure of the hypernuclei than in the models assuming simple 2-cluster potentials. In particular, we have found that the most recent quark-model interaction, the model fss2, yields a realistic description of many systems including the triton [17], hypertriton [18], ${}^{9}_{\Lambda}$Be and ${}^{6}_{\Lambda\Lambda}$He.

REFERENCES

1. Y. Fujiwara and R. Tamagaki, Prog. Theor. Phys. 56 (1976) 1503.
2. S. Oryu *et al.*, Few-Body systems 17 (1994) 185.
3. H. Kamada and S. Oryu, Prog. Theor. Phys. 76 (1986) 1260.
4. Y. Fujiwara *et al.*, Prog. Theor. Phys. 107 (2002) 745.
5. Y. Fujiwara *et al.*, Prog. Theor. Phys. 107 (2002) 993.
6. D. R. Thompson, M. LeMere and Y. C. Tang, Nucl. Phys. A286 (1977) 53.
7. Y. Fukushima and M. Kamimura, Suppl. to J. Phys. Soc. Jpn. 44 (1978) 225.
8. Y. Suzuki and K. Varga, private communication.
9. E. Uegaki *et al.*, Prog. Theor. Phys. 57 (1977) 1262.
10. E. W. Schmid, Z. Phys. A297 (1980) 105.
11. S. Oryu and H. Kamada, Nucl. Phys. A493 (1989) 91.
12. Y. Fujiwara, M. Kohno and Y. Suzuki, to be published in Few-Body Systems (2004).
13. Y. Fujiwara, M. Kohno, C. Nakamoto and Y. Suzuki, Phys. Rev. C64 (2001) 054001.
14. J.-M. Sparenberg and D. Baye, Phys. Rev. C55 (1997) 2175.
15. M. Kohno, Y. Fujiwara and Y. Akaishi, Phys. Rev. C68 (2003) 034302.
16. E. Hiyama *et al.*, Prog. Theor. Phys. 97 (1997) 881.
17. Y. Fujiwara *et al.*, Phys. Rev. C66 (2002) 021001 (R).
18. Y. Fujiwara, K. Miyagawa, M. Kohno and Y. Suzuki, contribution to this conference.