2004.5.5 Faddeev note No. 4 (revised by Y.F.)

§1. General

1.1. *3-Body Jacobi Coordinates*

We follow the same notation as in the Glöckle's textbook:

$(\alpha\beta\gamma)$ = a cyclic permutation of (123) and $M = m_\alpha + m_\beta + m_\gamma$

Coordinate representation

$$\begin{aligned}
&\boldsymbol{\rho}_\alpha = \boldsymbol{x}_\beta - \boldsymbol{x}_\gamma && \boldsymbol{x}_\alpha = \boldsymbol{X} + \tfrac{m_\beta+m_\gamma}{M}\boldsymbol{\lambda}_\alpha \\
&\boldsymbol{\lambda}_\alpha = \boldsymbol{x}_\alpha - \tfrac{m_\beta\boldsymbol{x}_\beta+m_\gamma\boldsymbol{x}_\gamma}{m_\beta+m_\gamma} && \boldsymbol{x}_\beta = \boldsymbol{X} - \tfrac{m_\alpha}{M}\boldsymbol{\lambda}_\alpha + \tfrac{m_\gamma}{m_\beta+m_\gamma}\boldsymbol{\rho}_\alpha \\
&\boldsymbol{X} = \tfrac{1}{M}\left(m_\alpha\boldsymbol{x}_\alpha + m_\beta\boldsymbol{x}_\beta + m_\gamma\boldsymbol{x}_\gamma\right) && \boldsymbol{x}_\gamma = \boldsymbol{X} - \tfrac{m_\alpha}{M}\boldsymbol{\lambda}_\alpha - \tfrac{m_\beta}{m_\beta+m_\gamma}\boldsymbol{\rho}_\alpha
\end{aligned} \tag{1·1}$$

Momentum coordinates

$$\begin{aligned}
&\boldsymbol{p}_\alpha = \tfrac{m_\gamma\boldsymbol{k}_\beta - m_\beta\boldsymbol{k}_\gamma}{m_\beta+m_\gamma} && \boldsymbol{k}_\alpha = \tfrac{m_\alpha}{M}\boldsymbol{K} + \boldsymbol{q}_\alpha \\
&\boldsymbol{q}_\alpha = \tfrac{1}{M}\left\{\left(m_\beta+m_\gamma\right)\boldsymbol{k}_\alpha - m_\alpha\left(\boldsymbol{k}_\beta+\boldsymbol{k}_\gamma\right)\right\} && \boldsymbol{k}_\beta = \tfrac{m_\beta}{M}\boldsymbol{K} - \tfrac{m_\beta}{m_\beta+m_\gamma}\boldsymbol{q}_\alpha + \boldsymbol{p}_\alpha \\
&\boldsymbol{K} = \boldsymbol{k}_\alpha + \boldsymbol{k}_\beta + \boldsymbol{k}_\gamma && \boldsymbol{k}_\gamma = \tfrac{m_\gamma}{M}\boldsymbol{K} - \tfrac{m_\gamma}{m_\beta+m_\gamma}\boldsymbol{q}_\alpha - \boldsymbol{p}_\alpha
\end{aligned} \tag{1·2}$$

Plane wave

$$e^{i\boldsymbol{p}_\alpha\boldsymbol{\rho}_\alpha + i\boldsymbol{q}_\alpha\boldsymbol{\lambda}_\alpha + i\boldsymbol{K}\boldsymbol{X}} = e^{i\boldsymbol{k}_1\boldsymbol{x}_1 + i\boldsymbol{k}_2\boldsymbol{x}_2 + i\boldsymbol{k}_3\boldsymbol{x}_3} \qquad \text{for} \qquad {}^\forall\alpha = 1,\ 2,\ 3 \tag{1·3}$$

Kinetic energies

$$\begin{aligned}
H_0 &= \frac{\hbar^2{\boldsymbol{k}_1}^2}{2m_1} + \frac{\hbar^2{\boldsymbol{k}_2}^2}{2m_2} + \frac{\hbar^2{\boldsymbol{k}_3}^2}{2m_3} - \frac{\hbar^2\boldsymbol{K}^2}{2M} = \frac{\hbar^2}{2\mu_\alpha}{\boldsymbol{p}_\alpha}^2 + \frac{\hbar^2}{2\mu_{\bar\alpha}}{\boldsymbol{q}_\alpha}^2 \\
&\text{for} \qquad {}^\forall\alpha = 1,\ 2,\ 3 \\
&\text{with} \qquad \mu_\alpha = \frac{m_\beta m_\gamma}{m_\beta+m_\gamma} \qquad \textit{and} \qquad \mu_{\bar\alpha} = \frac{1}{M}m_\alpha\left(m_\beta+m_\gamma\right)
\end{aligned} \tag{1·4}$$

Coordinate transformation in configuration space

$$\begin{pmatrix} \boldsymbol{\rho}_\beta \\ \boldsymbol{\lambda}_\beta \end{pmatrix} = \begin{pmatrix} -\frac{m_\beta}{m_\beta+m_\gamma} & -1 \\ \frac{m_\gamma M}{\left(m_\beta+m_\gamma\right)\left(m_\alpha+m_\gamma\right)} & -\frac{m_\alpha}{m_\alpha+m_\gamma} \end{pmatrix} \begin{pmatrix} \boldsymbol{\rho}_\alpha \\ \boldsymbol{\lambda}_\alpha \end{pmatrix} \tag{1·5}$$

$$\begin{pmatrix} \boldsymbol{\rho}_\alpha \\ \boldsymbol{\lambda}_\alpha \end{pmatrix} = \begin{pmatrix} -\frac{m_\alpha}{m_\alpha+m_\gamma} & 1 \\ -\frac{m_\gamma M}{\left(m_\beta+m_\gamma\right)\left(m_\alpha+m_\gamma\right)} & -\frac{m_\beta}{m_\beta+m_\gamma} \end{pmatrix} \begin{pmatrix} \boldsymbol{\rho}_\beta \\ \boldsymbol{\lambda}_\beta \end{pmatrix} \tag{1·6}$$

Coordinate transformation in momentum space

$$\begin{pmatrix} \boldsymbol{p}_\beta \\ \boldsymbol{q}_\beta \end{pmatrix} = \begin{pmatrix} -\frac{m_\alpha}{m_\alpha+m_\gamma} & -\frac{m_\gamma M}{\left(m_\beta+m_\gamma\right)(m_\alpha+m_\gamma)} \\ 1 & -\frac{m_\beta}{m_\beta+m_\gamma} \end{pmatrix} \begin{pmatrix} \boldsymbol{p}_\alpha \\ \boldsymbol{q}_\alpha \end{pmatrix} \tag{1·7}$$

$$\begin{pmatrix} \boldsymbol{p}_\alpha \\ \boldsymbol{q}_\alpha \end{pmatrix} = \begin{pmatrix} -\frac{m_\beta}{m_\beta+m_\gamma} & \frac{m_\gamma M}{\left(m_\beta+m_\gamma\right)(m_\alpha+m_\gamma)} \\ -1 & -\frac{m_\alpha}{m_\alpha+m_\gamma} \end{pmatrix} \begin{pmatrix} \boldsymbol{p}_\beta \\ \boldsymbol{q}_\beta \end{pmatrix} \tag{1·8}$$

Scaling

$$\begin{cases} \widehat{\boldsymbol{\rho}_\alpha} = \sqrt{\mu_\alpha}\boldsymbol{\rho}_\alpha \\ \widehat{\boldsymbol{\lambda}_\alpha} = \sqrt{\mu_{\bar{\alpha}}}\boldsymbol{\lambda}_\alpha \end{cases} , \qquad \begin{cases} \widehat{\boldsymbol{p}_\alpha} = \frac{1}{\sqrt{\mu_\alpha}}\boldsymbol{p}_\alpha \\ \widehat{\boldsymbol{q}_\alpha} = \frac{1}{\sqrt{\mu_{\bar{\alpha}}}}\boldsymbol{q}_\alpha \end{cases} \tag{1·9}$$

The orthogonal matrix

$$P^{(\beta\alpha)} = \frac{1}{\sqrt{\left(m_\beta+m_\gamma\right)\left(m_\alpha+m_\gamma\right)}} \begin{pmatrix} -\sqrt{m_\beta m_\alpha} & -\sqrt{m_\gamma M} \\ \sqrt{m_\gamma M} & -\sqrt{m_\beta m_\alpha} \end{pmatrix} \tag{1·10}$$

with

$${}^tP^{(\beta\alpha)}\,P^{(\beta\alpha)} = P^{(\beta\alpha)}\,{}^tP^{(\beta\alpha)} = 1 \tag{1·11}$$

can be used both in the coordinate and momentum representations:

$$\begin{pmatrix} \widehat{\boldsymbol{\rho}}_\beta \\ \widehat{\boldsymbol{\lambda}}_\beta \end{pmatrix} = P^{(\beta\alpha)} \begin{pmatrix} \widehat{\boldsymbol{\rho}}_\alpha \\ \widehat{\boldsymbol{\lambda}}_\alpha \end{pmatrix} \qquad , \qquad \begin{pmatrix} \widehat{\boldsymbol{p}}_\beta \\ \widehat{\boldsymbol{q}}_\beta \end{pmatrix} = P^{(\beta\alpha)} \begin{pmatrix} \widehat{\boldsymbol{p}}_\alpha \\ \widehat{\boldsymbol{q}}_\alpha \end{pmatrix} \tag{1·12}$$

The inverse transformation is accomplished by ${}^tP^{(\beta\alpha)}$ (The off-diagonal elements change the sign.)

Invariance of integral volumes

$$d\boldsymbol{p}_\alpha d\boldsymbol{q}_\alpha = d\boldsymbol{p}_\beta d\boldsymbol{q}_\beta = d\boldsymbol{p}_\gamma d\boldsymbol{q}_\gamma \tag{1·13}$$

since

$$d\boldsymbol{p}_\alpha \wedge d\boldsymbol{q}_\alpha \wedge d\boldsymbol{K} = -d\boldsymbol{k}_\alpha \wedge d\boldsymbol{k}_\beta \wedge d\boldsymbol{k}_\gamma \tag{1·14}$$

1.2. *Dirac notation*

We choose $\gamma = 3$ coordinates as the standard set of the Jacobi coordinates, and introduce the Dirac notation:

$$
\begin{aligned}
&|\boldsymbol{p}_\gamma \boldsymbol{q}_\gamma; \alpha\beta\gamma\rangle = |\boldsymbol{k}_\alpha, \boldsymbol{k}_\beta, \boldsymbol{k}_\gamma; \alpha\beta\gamma\rangle \\
&\boldsymbol{p}_\gamma = \frac{m_\beta \boldsymbol{k}_\alpha - m_\alpha \boldsymbol{k}_\beta}{m_\alpha + m_\beta} \\
&\boldsymbol{q}_\gamma = \frac{1}{m_\alpha + m_\beta + m_\gamma} \left[(m_\alpha + m_\beta)\boldsymbol{k}_\gamma - m_\gamma (\boldsymbol{k}_\alpha + \boldsymbol{k}_\beta) \right] \quad ,
\end{aligned}
\tag{1·15}
$$

where $\alpha\beta\gamma$ is a *cyclic* permutation of 123. For an arbitrary function $f(\boldsymbol{pq}; 123) = \langle \boldsymbol{p}_\gamma \boldsymbol{q}_\gamma; \alpha\beta\gamma | f \rangle$, the effects of the permutation $P_{(123)} \in S_3$ etc. are

$$
\begin{aligned}
&P_{(123)} f(\boldsymbol{p}_2, \boldsymbol{q}_2; 312) = {P_{(123)}}^2 f(\boldsymbol{p}_1, \boldsymbol{q}_1; 231) = f(\boldsymbol{p}_3, \boldsymbol{q}_3; 123) \\
&P_{(12)} f(\boldsymbol{p}_3, \boldsymbol{q}_3; 123) = f(-\boldsymbol{p}_3, \boldsymbol{q}_3; 213) \\
&P_{(12)} f(\boldsymbol{p}_1, \boldsymbol{q}_1; 231) = f(-\boldsymbol{p}_2, \boldsymbol{q}_2; 132) \\
&P_{(12)} f(\boldsymbol{p}_2, \boldsymbol{q}_2; 312) = f(-\boldsymbol{p}_1, \boldsymbol{q}_1; 321)
\end{aligned}
\tag{1·16}
$$

If we combine these, we also obtain

$$
\begin{aligned}
&P_{(23)} f(\boldsymbol{p}_3, \boldsymbol{q}_3; 123) = P_{(13)} f(\boldsymbol{p}_2, \boldsymbol{q}_2; 312) = f(-\boldsymbol{p}_2, \boldsymbol{q}_2; 132) \\
&P_{(13)} f(\boldsymbol{p}_3, \boldsymbol{q}_3; 123) = P_{(23)} f(\boldsymbol{p}_1, \boldsymbol{q}_1; 231) = f(-\boldsymbol{p}_1, \boldsymbol{q}_1; 321)
\end{aligned}
\tag{1·17}
$$

From the rule in Eq. (1·15), we can easily see that it is actually redundant to write $\boldsymbol{p}_\gamma \boldsymbol{q}_\gamma$ and that the effects of the permutation are known only from the variation of $f(\alpha\beta\gamma)$, namely, the exchange of the spin-isospin variables. If we omit the spin and isospin degrees of freedom, we find

$$
P_{(12)} f(\boldsymbol{p}_1 \boldsymbol{q}_1) = P_{(23)} f(\boldsymbol{p}_3 \boldsymbol{q}_3) = P_{(13)} f(\boldsymbol{p}_2 \boldsymbol{q}_2) = f(-\boldsymbol{p}_2 \boldsymbol{q}_2) \tag{1·18}
$$

Similarly, we find

$$
P_{(12)} f(\boldsymbol{p}_2 \boldsymbol{q}_2) = P_{(13)} f(\boldsymbol{p}_3 \boldsymbol{q}_3) = P_{(23)} f(\boldsymbol{p}_1 \boldsymbol{q}_1) = f(-\boldsymbol{p}_1 \boldsymbol{q}_1) \tag{1·19}
$$

As an application, we can show

$$
\begin{aligned}
&{P_{(123)}}^2 v(\boldsymbol{p}_3 \boldsymbol{q}_3) = v(\boldsymbol{p}_2 \boldsymbol{q}_2) \\
&P_{(12)} {P_{(123)}}^2 v(\boldsymbol{p}_3 \boldsymbol{q}_3) = P_{(13)} v(\boldsymbol{p}_3 \boldsymbol{q}_3) = v(-\boldsymbol{p}_1 \boldsymbol{q}_1)
\end{aligned}
\tag{1·20}
$$

In the following, we will write these as ${P_{(123)}}^2 v$ and $P_{(12)} {P_{(123)}}^2 v$, respectively.

1.3. *Reduction of independent functions*

We can reduce the number of independent functions for the Faddeev equation, if some of the particles are identical. Let us first write the Faddeev equation as

$$\begin{aligned}
\Psi &= \psi^{(12)} + \psi^{(23)} + \psi^{(31)} \\
\psi^{(12)} &= G_0 t_{12} \left(\psi^{(23)} + \psi^{(31)}\right) \\
\psi^{(23)} &= G_0 t_{23} \left(\psi^{(31)} + \psi^{(12)}\right) \\
\psi^{(31)} &= G_0 t_{31} \left(\psi^{(12)} + \psi^{(23)}\right)
\end{aligned} \tag{1·21}$$

3 identical particles

We assume

$$\begin{aligned}
\Psi &= \psi^{(12)} + \psi^{(23)} + \psi^{(31)} \\
&= \psi + P_{(123)}\psi + {P_{(123)}}^2\psi \qquad \text{with} \qquad P_{(12)}\psi = \pm\psi
\end{aligned} \tag{1·22}$$

The sign $\pm$ in $P_{(12)}\psi = \pm\psi$ corresponds to the boson and the fermion systems. The three equations become identical:

$$\psi = G_0 t(P_{(123)} + {P_{(123)}}^2)\psi \qquad \text{or} \qquad \psi = G_0 t(1 \pm P_{(12)}){P_{(123)}}^2\psi \tag{1·23}$$

Here, $t = t_{12}$.

2 identical particles

We can assume

$$\begin{aligned}
\Psi &= \psi^{(12)} + \psi^{(23)} + \psi^{(31)} \\
&= u \pm P_{(12)}{P_{(123)}}^2 v + {P_{(123)}}^2 v \\
&\quad \text{with} \qquad P_{(12)}\psi^{(12)} = \pm\psi^{(12)} \qquad \text{or} \qquad P_{(12)}u = \pm u
\end{aligned} \tag{1·24}$$

Then, we get two equations

$$\begin{aligned}
\psi^{(12)} &= G_0 t_{12}(1 \pm P_{(12)})\psi^{(31)} \\
\psi^{(31)} &= G_0 t_{31} \left(\psi^{(12)} \pm P_{(12)}\psi^{(31)}\right)
\end{aligned} \tag{1·25}$$

or

$$\begin{aligned}
u &= G_0 t(1 \pm P_{(12)}){P_{(123)}}^2 v \\
v &= G_0 \tau \left(P_{(123)}u \pm P_{(23)}v\right)
\end{aligned} \tag{1·26}$$

Here. $t = t_{12}$ and $\tau = P_{(123)} t_{13} {P_{(123)}}^{-1}$.

Momentum representation

The total wave function is explicitly expressed as

$$\begin{aligned}\Psi(\boldsymbol{p}_3\boldsymbol{q}_3) &= u(\boldsymbol{p}_3\boldsymbol{q}_3) \pm P_{(12)}{P_{(123)}}^2 v(\boldsymbol{p}_3\boldsymbol{q}_3) + {P_{(123)}}^2 v(\boldsymbol{p}_3\boldsymbol{q}_3) \\ &= u(\boldsymbol{p}_3, \boldsymbol{q}_3) \pm v(-\boldsymbol{p}_1, \boldsymbol{q}_1) + v(\boldsymbol{p}_2, \boldsymbol{q}_2) \\ &\quad \text{with} \qquad u(-\boldsymbol{p}_3, \boldsymbol{q}_3) = \pm u(\boldsymbol{p}_3, \boldsymbol{q}_3) \quad , \end{aligned} \tag{1·27}$$

which satisfies the symmetry requirement

$$P_{12}\Psi(\boldsymbol{p}_3\boldsymbol{q}_3) = \pm\Psi(\boldsymbol{p}_3\boldsymbol{q}_3) \qquad \text{or} \qquad \Psi(-\boldsymbol{p}_3, \boldsymbol{q}_3) = \pm\Psi(\boldsymbol{p}_3, \boldsymbol{q}_3) \tag{1·28}$$

The Faddeev equation in the momentum representation is straightforwardly derived. To derive the second equation, we go back to Eq. (1·25) and use the fact that $t_{13} = {P_{(123)}}^2 \tau {P_{(123)}}^{-2}$ actually implies

$$\begin{aligned}\langle \boldsymbol{p}_3\boldsymbol{q}_3|t_{13}|\boldsymbol{p}_3{}'\boldsymbol{q}_3{}'\rangle &= {P_{(123)}}^2 \langle \boldsymbol{p}_3\boldsymbol{q}_3|\tau|\boldsymbol{p}_3{}'\boldsymbol{q}_3{}'\rangle {P_{(123)}}^{-2} \\ &= \langle \boldsymbol{p}_2\boldsymbol{q}_2|\tau|\boldsymbol{p}_2{}'\boldsymbol{q}_2{}'\rangle \\ &= \langle \boldsymbol{p}_2|\tau|\boldsymbol{p}_2{}'\rangle \ \delta(\boldsymbol{q}_2 - \boldsymbol{q}_2{}') \end{aligned} \tag{1·29}$$

We obtain

$$\begin{aligned} u(\boldsymbol{p}, \boldsymbol{q}_3) &= G_0(\boldsymbol{p}\boldsymbol{q}_3) \int d\boldsymbol{p}_3 \langle \boldsymbol{p}|t(E - h_{\bar{3}})|\boldsymbol{p}_3\rangle \ [v(\boldsymbol{p}_2, \boldsymbol{q}_2) \pm v(-\boldsymbol{p}_1, \boldsymbol{q}_1)] \\ v(\boldsymbol{p}, \boldsymbol{q}_2) &= G_0(\boldsymbol{p}\boldsymbol{q}_2) \int d\boldsymbol{p}_2 \langle \boldsymbol{p}|\tau(E - h_{\bar{2}})|\boldsymbol{p}_2\rangle \ [u(\boldsymbol{p}_3, \boldsymbol{q}_3) \pm v(-\boldsymbol{p}_1, \boldsymbol{q}_1)] \quad , \end{aligned} \tag{1·30}$$

where $\boldsymbol{p}_3$ $\boldsymbol{q}_3$, $\boldsymbol{p}_2$ $\boldsymbol{q}_2$, $\boldsymbol{p}_1$ $\boldsymbol{q}_1$ are related to each other in the standard relationship.

1.4. *Transformation coefficients for 3-body rearrangements*

We calculate the rearrangement factors by taking $\langle\psi|{P_{(123)}}^2|\varphi\rangle$ as an example. From the definition of $P_{(123)}$, we find

$$\begin{aligned}&\langle \boldsymbol{p}_3, \boldsymbol{q}_3; 123|{P_{(123)}}^2\varphi\rangle = {P_{(123)}}^2 \varphi(\boldsymbol{p}_3, \boldsymbol{q}_3; 123) = \varphi(\boldsymbol{p}_2, \boldsymbol{q}_2; 312) \\ &= \int d\boldsymbol{p}' d\boldsymbol{q}' \ \delta(\boldsymbol{p}' - \boldsymbol{p}_2) \ \delta(\boldsymbol{q}' - \boldsymbol{q}_2) {P_{(123)}^{(\sigma\tau)}}^2 \varphi(\boldsymbol{p}', \boldsymbol{q}'; 123) \ , \end{aligned} \tag{1·31}$$

where $P_{(123)}^{(\sigma\tau)}$ operates only on the spin-isospin variables of $\varphi(\boldsymbol{p}', \boldsymbol{q}'; 123)$. We write $\boldsymbol{p}_3$, $\boldsymbol{q}_3$ as $\boldsymbol{p}$, $\boldsymbol{q}$, and make the partial wave decomposition through

$$\begin{aligned}\langle \boldsymbol{p}, \boldsymbol{q}; 123|{P_{(123)}}^2\varphi\rangle &= \sum_\gamma \langle \widehat{\boldsymbol{p}}, \widehat{\boldsymbol{q}}; 123|\gamma\rangle \ \langle p, q, \gamma|{P_{(123)}}^2\varphi\rangle, \\ \psi(\boldsymbol{p}', \boldsymbol{q}'; 123) &= \sum_\beta \langle \widehat{\boldsymbol{p}}', \widehat{\boldsymbol{q}}'; 123|\beta\rangle \ \varphi_\beta(p', q') \ . \end{aligned} \tag{1·32}$$

Here, the summations are over all the angular-momentum (and also spin, isospin) quantum numbers in the γ or β channel. Then we find

$$\langle p,q,\gamma|{P_{(123)}}^2\varphi\rangle = \sum_\beta \int_0^\infty {p'}^2 dp' \int_0^\infty {q'}^2 dq' \ \langle p,q,\gamma|{P_{(123)}}^2|p',q',\beta\rangle_{3-2} \ \varphi_\beta(p',q') , \quad (1{\cdot}33)$$

$$\langle p,q,\gamma|{P_{(123)}}^2|p',q',\beta\rangle_{3-2} = \sum_{123} \int d\widehat{\boldsymbol{p}} d\widehat{\boldsymbol{q}} d\widehat{\boldsymbol{p}}' d\widehat{\boldsymbol{q}}' \ \langle\gamma|\widehat{\boldsymbol{p}},\widehat{\boldsymbol{q}};123\rangle \delta\left(\boldsymbol{p}'-\boldsymbol{p}_2\right)\delta\left(\boldsymbol{q}'-\boldsymbol{q}_2\right)$$
$$\times {P_{(123)}^{(\sigma\tau)}}^2 \langle\widehat{\boldsymbol{p}}',\widehat{\boldsymbol{q}}';123|\beta\rangle \ . \quad (1{\cdot}34)$$

Here, $\boldsymbol{p}_2$ and $\boldsymbol{q}_2$ are obtained from Eq. (1·8) by setting $\alpha = 2$, $\beta = 3$, and $\boldsymbol{p}_\beta = \boldsymbol{p}$, $\boldsymbol{q}_\beta = \boldsymbol{q}$. This implies Eq. (1·34) being explicitly given by

$$\langle p,q,\gamma|{P_{(123)}}^2|p',q',\beta\rangle_{3-2} = \sum_{123} \int d\widehat{\boldsymbol{p}} d\widehat{\boldsymbol{q}} d\widehat{\boldsymbol{p}}' d\widehat{\boldsymbol{q}}' \ \langle\gamma|\widehat{\boldsymbol{p}},\widehat{\boldsymbol{q}};123\rangle \ \delta\left(\boldsymbol{p}+\boldsymbol{q}'+\frac{m_2}{m_2+m_1}\boldsymbol{q}\right)$$
$$\times\delta\left(\boldsymbol{p}'-\boldsymbol{q}-\frac{m_3}{m_3+m_1}\boldsymbol{q}'\right) \ {P_{(123)}^{(\sigma\tau)}}^2 \ \langle\widehat{\boldsymbol{p}}',\widehat{\boldsymbol{q}}';123|\beta\rangle \ . \quad (1{\cdot}35)$$

We define

$$\boldsymbol{p}_1 = -\left(\boldsymbol{q}'+\frac{m_2}{m_2+m_1}\boldsymbol{q}\right) \quad , \qquad \boldsymbol{p}_2 = \boldsymbol{q}+\frac{m_3}{m_3+m_1}\boldsymbol{q}' \ , \quad (1{\cdot}36)$$

and follow the standard procedure. For the spin-angular (and also isospin) functions, we further separate the orbital angular-momentum part through

$$\begin{aligned}\langle\widehat{\boldsymbol{p}},\widehat{\boldsymbol{q}};123|\gamma\rangle &= \sum_{LM} \ \langle\widehat{\boldsymbol{p}},\widehat{\boldsymbol{q}}|(\lambda\ell)LM\rangle \ \langle(\lambda\ell)LM;123|\gamma\rangle \\ &= \sum_{LM} Y_{(\lambda\ell)LM}(\widehat{\boldsymbol{p}},\widehat{\boldsymbol{q}}) \ \langle(\lambda\ell)LM;123|\gamma\rangle \ , \\ \langle\widehat{\boldsymbol{p}}',\widehat{\boldsymbol{q}}';123|\beta\rangle &= \sum_{LM} Y_{(\ell_1,\ell_2)LM}(\widehat{\boldsymbol{p}}',\widehat{\boldsymbol{q}}') \ \langle(\ell_1\ell_2)LM;123|\beta\rangle \ . \end{aligned} \quad (1{\cdot}37)$$

Then we find

$$\langle p,q,\gamma|{P_{(123)}}^2|p',q',\beta\rangle_{3-2} = \frac{1}{2}\int_{-1}^1 dx \frac{\delta(p-p_1)}{p^{\lambda+2}} \frac{\delta(p'-p_2)}{{p'}^{\ell_1+2}} g_{\gamma\beta}(q,q';x) \ , \quad (1{\cdot}38)$$

$$\langle p,q,\gamma|{P_{(123)}}^2\varphi\rangle = \frac{1}{2}\sum_\beta \int_0^\infty {q'}^2 dq' \int_{-1}^1 dx \frac{\delta(p-p_1)}{p^{\lambda+2}} g_{\gamma\beta}(q,q';x) \frac{1}{{p_2}^{\ell_1}} \ \varphi_\beta(p_2,q'), \quad (1{\cdot}39)$$

where

$$p_1 = p\left(q',\frac{m_2}{m_2+m_1}q;x\right) \quad , \qquad p_2 = p\left(q,\frac{m_3}{m_3+m_1}q';x\right) \ , \quad (1{\cdot}40)$$

with $p(q,q';x)=\sqrt{q^2+q'^2+2qq'x}$. The rearrangement factor $g_{\gamma\beta}(q,q';x)$ is given by

$$g_{\gamma\beta}(q,q';x) = \sum_{\lambda_1+\lambda_2=\lambda,\ \lambda_1'+\lambda_2'=\ell_1} q^{\lambda_1'+\lambda_2} q'^{\lambda_1+\lambda_2'} \left(\frac{m_2}{m_2+m_1}\right)^{\lambda_2} \left(\frac{m_3}{m_3+m_1}\right)^{\lambda_2'} \times \sum_k (2k+1)\ g_{\gamma\beta}^{\lambda_1\lambda_1' k}\ P_k(x)\ , \tag{1·41}$$

$$g_{\gamma\beta}^{\lambda_1\lambda_1' k} = (-1)^{\lambda} \sum_{LM}\sum_{123} \langle\gamma|(\lambda\ell)LM;123\rangle P_{(123)}^{(\sigma\tau)\,2} \langle((\ell_1\ell_2)LM;123|\beta\rangle\ G_{(\lambda\ell),(\ell_1\ell_2)}^{\lambda_1\lambda_1' kL}\ , \tag{1·42}$$

with the spatial rearrangement factor

$$\begin{aligned} G_{(\lambda\ell),(\ell_1\ell_2)}^{\lambda_1\lambda_1' kL} &= G_{(\ell_1\ell_2),(\lambda\ell)}^{\lambda_1'\lambda_1 kL} = 4\pi \left[\frac{(2\lambda+1)!(2\ell_1+1)!}{(2\lambda_1+1)!(2\lambda_2+1)!(2\lambda_1'+1)!(2\lambda_2'+1)!}\right]^{\frac{1}{2}} \\ &\quad\times \int d\widehat{\boldsymbol{q}} d\widehat{\boldsymbol{q}}' \left[Y_{(\lambda_1\lambda_2)\lambda}(\widehat{\boldsymbol{q}}',\widehat{\boldsymbol{q}}) Y_\ell(\widehat{\boldsymbol{q}})\right]_{LM}^{*} P_k(\widehat{\boldsymbol{q}}\cdot\widehat{\boldsymbol{q}}') \left[Y_{(\lambda_1'\lambda_2')\ell_1}(\widehat{\boldsymbol{q}},\widehat{\boldsymbol{q}}') Y_{\ell_2}(\widehat{\boldsymbol{q}}')\right]_{LM} \\ &= \left[\frac{(2\lambda+1)!(2\ell_1+1)!}{(2\lambda_1)!(2\lambda_2)!(2\lambda_1')!(2\lambda_2')!}\right]^{\frac{1}{2}} \widehat{\lambda}\widehat{\ell}\widehat{\ell_1}\widehat{\ell_2} \sum_{f,f'} \langle\lambda_2 0\ell 0|f0\rangle\langle\lambda_2'0\ell_2 0|f'0\rangle\langle k0\lambda_1 0|f'0\rangle \\ &\quad\times\langle k0\lambda_1'0|f0\rangle \begin{Bmatrix} f & L & \lambda_1 \\ \lambda & \lambda_2 & \ell \end{Bmatrix} \begin{Bmatrix} f' & L & \lambda_1' \\ \ell_1 & \lambda_2' & \ell_2 \end{Bmatrix} \begin{Bmatrix} \lambda_1' & f' & L \\ \lambda_1 & f & k \end{Bmatrix}\ . \end{aligned} \tag{1·43}$$

Here, $\widehat{\lambda}=\sqrt{2\lambda+1}$ etc. and $\lambda_2=\lambda-\lambda_1$, $\lambda_2'=\ell_1-\lambda_1'$ with $\lambda_1=0\sim\lambda$, $\lambda_1'=0\sim\ell_1$. For the spin-isospin part of Eq. (1·42), some angular-momentum recouplings yield

$$\begin{aligned} g_{\gamma\beta}^{\lambda_1\lambda_1' k} &= (-1)^{\lambda}\ G_{(\lambda\ell),(\ell_1\ell_2)}^{\lambda_1\lambda_1' kL}\ \langle STT_z;\gamma||P_{(123)}^{(\sigma\tau)\,2}||STT_z;\beta\rangle_{\text{spin}-\text{isospin}} \qquad (LS-\text{coupling})\ , \\ &= (-1)^{\lambda} \sum_{LS} \begin{bmatrix} \lambda & s_1 & I \\ \ell & s_2 & j \\ L & S & J \end{bmatrix} \begin{bmatrix} \ell_1 & s_1' & j_1 \\ \ell_2 & s_2' & j_2 \\ L & S & J \end{bmatrix} G_{(\lambda\ell),(\ell_1\ell_2)}^{\lambda_1\lambda_1' kL} \\ &\quad\times\langle STT_z;\gamma||P_{(123)}^{(\sigma\tau)\,2}||STT_z;\beta\rangle_{\text{spin}-\text{isospin}} \qquad (jj-\text{coupling})\ , \end{aligned} \tag{1·44}$$

where the square bracket implies the unitary form of the $9j$ coefficients and the quantum numbers are specified by

$$\begin{aligned} &|\gamma\rangle = |[(\lambda\ell)LS]JJ_z;TT_z\rangle, \quad |\beta\rangle = |[(\ell_1\ell_2)LS]JJ_z;TT_z\rangle \qquad (LS-\text{coupling})\ , \\ &|\gamma\rangle = |[(\lambda s_1)I(\ell s_2)j]JJ_z;TT_z\rangle, \quad |\beta\rangle = |[(\ell_1 s_1')j_1(\ell_2 s_2')j_2]JJ_z;TT_z\rangle \quad (jj-\text{coupling})\ . \end{aligned} \tag{1·45}$$

Note that the permutation operator $P_{(123)}^{(\sigma\tau)\,2}$ does not change the total spin and isospin values, S and TT_z.

The other types of rearrangement factors are obtained in a similar way. First, the symmetry of the matrix elements yields

$$\langle p,q,\beta|P_{(123)}|p',q',\gamma\rangle_{2-3} = \langle p',q',\gamma|P_{(123)}{}^2|p,q,\beta\rangle_{3-2}\ . \tag{1·46}$$

The rearrangement factor for the matrix element $\langle\varphi|P_{(23))}|\varphi\rangle$ needs a little care, since the mass assignment of the three particles is made in the standard Jacobi coordinates $\gamma = 3$. We first use $P_{(23)} = P_{(123)}P_{(12)}{P_{(123)}}^{-1}$ and write the matrix element as

$$\langle\varphi|P_{(23)}|\varphi\rangle = \sum_{123}\int d\boldsymbol{p}_3 d\boldsymbol{q}_3\ \varphi^*(\boldsymbol{p}_2,\boldsymbol{q}_2;312)\ \varphi(-\boldsymbol{p}_1,\boldsymbol{q}_1;321)\ . \tag{1·47}$$

The corresponding rearrangement factor in the Dirac notation is given by

$$\langle p,q,\beta|P_{(123)}|p',q',\beta'\rangle_{2-1} = \sum_{123}\int d\widehat{\boldsymbol{p}}d\widehat{\boldsymbol{q}}d\widehat{\boldsymbol{p}}'d\widehat{\boldsymbol{q}}'\ \langle\beta|\widehat{\boldsymbol{p}},\widehat{\boldsymbol{q}};123\rangle\ \delta\left(\boldsymbol{p}+\boldsymbol{q}'+\frac{m_1}{m_1+m_3}\boldsymbol{q}\right)$$
$$\times\delta\left(\boldsymbol{p}'+\boldsymbol{q}+\frac{m_2}{m_2+m_3}\boldsymbol{q}'\right)\ P^{(\sigma\tau)}_{(23)}\ \langle\widehat{\boldsymbol{p}}',\widehat{\boldsymbol{q}}';123|\beta'\rangle\ . \tag{1·48}$$

Note that the rearrangement factor Eq. (1·48) is symmetric with respect to the interchange between $p,\ q,\ \beta$ and $p',\ q',\ \beta'$, since $m_1 = m_2$.

Table I. Three-body systems with two identical particles

particle	mass	${}^{9}_{\Lambda}$Be	${}^{6}_{\Lambda\Lambda}$He	${}^{3}_{\Lambda}$H	^{3}H	^{3}He
1	m_1	α_1	Λ_1	N	n	p
2	m_2	α_2	Λ_2	N	n	p
3	m_3	Λ	α	Λ or Σ	p	n

§2. $\alpha\alpha\Lambda$ system

For 3-particle systems with two identical particles, we assign particle numbers 1, 2, 3 as in Table 1 with $m_1 = m_2$ and fix $(\alpha\beta\gamma) = (123)$. For ${}^{9}_{\Lambda}$Be, channels are specified as

(γ channel) $\gamma = 3$ with $(\gamma\alpha\beta) = (312)$

$$\boldsymbol{p}_3 = \frac{1}{2}(\boldsymbol{k}_1 - \boldsymbol{k}_2)$$
$$\boldsymbol{q}_3 = \frac{1}{8+\zeta}\left[8\boldsymbol{k}_3 - \zeta(\boldsymbol{k}_1+\boldsymbol{k}_2)\right] \tag{2·1}$$

where $\zeta = M_\Lambda/M_N$. The partial wave decomposition is specified by

$$|p_3q_3\gamma\rangle = |p_3q_3;[(\lambda\ell)L1/2]JJ_z\rangle \qquad \text{with} \qquad \gamma = (\lambda\ell)L1/2;JJ_z$$
$$\langle\widehat{\boldsymbol{p}}_3\widehat{\boldsymbol{q}}_3|\gamma\rangle = \left[Y_{(\lambda\ell)L}(\widehat{\boldsymbol{p}}_3,\widehat{\boldsymbol{q}}_3)\chi_{\frac{1}{2}}(3)\right]_{JJ_z}\phi(\alpha_1)\phi(\alpha_2) \tag{2·2}$$

in the LS-coupling scheme and

$$
\begin{aligned}
&|p_3 q_3 \gamma\rangle = |p_3 q_3; [\lambda(\ell 1/2) j] J J_z\rangle \qquad \text{with} \qquad \gamma = \lambda(\ell 1/2) j; J J_z \\
&\langle \widehat{\boldsymbol{p}}_3 \widehat{\boldsymbol{q}}_3 | \gamma\rangle = \left[Y_\lambda(\widehat{\boldsymbol{p}}_3) \left[Y_\ell(\widehat{\boldsymbol{q}}_3) \chi_{\frac{1}{2}}(3) \right]_j \right]_{JJ_z} \phi(\alpha_1)\phi(\alpha_2)
\end{aligned}
\tag{2·3}
$$

in the jj-coupling scheme. Similarly, we define

(β channel) $\beta = 2$ with $(\beta\gamma\alpha) = (231)$

$$
\begin{aligned}
\boldsymbol{p}_2 &= \frac{1}{4+\zeta}(4\boldsymbol{k}_3 - \zeta \boldsymbol{k}_1) \\
\boldsymbol{q}_2 &= \frac{1}{8+\zeta}\left[(4+\zeta)\boldsymbol{k}_2 - 4(\boldsymbol{k}_3 + \boldsymbol{k}_1)\right]
\end{aligned}
\tag{2·4}
$$

$$
\begin{aligned}
&|p_2 q_2 \beta\rangle = |p_2 q_2; [(\ell_1\ell_2) L 1/2] J J_z\rangle \qquad \text{with} \qquad \beta = (\ell_1\ell_2) L 1/2; J J_z \\
&\langle \widehat{\boldsymbol{p}}_2 \widehat{\boldsymbol{q}}_2 | \beta\rangle = \left[Y_{(\ell_1\ell_2)L}(\widehat{\boldsymbol{p}}_2, \widehat{\boldsymbol{q}}_2) \chi_{\frac{1}{2}}(3) \right]_{JJ_z} \phi(\alpha_1)\phi(\alpha_2) \\
&\qquad\qquad \text{in } LS\text{−coupling} \\
&|p_2 q_2 \beta\rangle = |p_2 q_2; [(\ell_1 1/2) I \ell_2] J J_z\rangle \qquad \text{with} \qquad \beta = (\ell_1 1/2) I \ell_2; J J_z \\
&\langle \widehat{\boldsymbol{p}}_2 \widehat{\boldsymbol{q}}_2 | \beta\rangle = \left[\left[Y_{\ell_1}(\widehat{\boldsymbol{p}}_2) \chi_{\frac{1}{2}}(3) \right]_I Y_{\ell_2}(\widehat{\boldsymbol{q}}_2) \right]_{JJ_z} \phi(\alpha_1)\phi(\alpha_2) \\
&\qquad\qquad \text{in } jj\text{−coupling}
\end{aligned}
\tag{2·5}
$$

(α channel) $\alpha = 1$ with $(\alpha\beta\gamma) = (123)$

$$
\begin{aligned}
\boldsymbol{p}_1 &= \frac{1}{4+\zeta}(\zeta \boldsymbol{k}_2 - 4\boldsymbol{k}_3) \\
\boldsymbol{q}_1 &= \frac{1}{8+\zeta}\left[(4+\zeta)\boldsymbol{k}_1 - 4(\boldsymbol{k}_2 + \boldsymbol{k}_3)\right]
\end{aligned}
\tag{2·6}
$$

For the quantum numbers, we use the same notation as that of the β channel.

$$
\begin{aligned}
&|p_1 q_1 \beta\rangle = |p_1 q_1; [(\ell_1\ell_2) L 1/2] J J_z\rangle \qquad \text{with} \qquad \beta = (\ell_1\ell_2) L 1/2; J J_z \\
&\langle \widehat{\boldsymbol{p}}_1 \widehat{\boldsymbol{q}}_1 | \beta\rangle = \left[Y_{(\ell_1\ell_2)L}(\widehat{\boldsymbol{p}}_1, \widehat{\boldsymbol{q}}_1) \chi_{\frac{1}{2}}(3) \right]_{JJ_z} \phi(\alpha_1)\phi(\alpha_2) \\
&\qquad\qquad \text{in } LS\text{−coupling} \\
&|p_1 q_1 \alpha\rangle = |p_1 q_1; [(\ell_1 1/2) I \ell_2] J J_z\rangle \qquad \text{with} \qquad \alpha = (\ell_1 1/2) I \ell_2; J J_z \\
&\langle \widehat{\boldsymbol{p}}_1 \widehat{\boldsymbol{q}}_1 | \alpha\rangle = \left[\left[Y_{\ell_1}(\widehat{\boldsymbol{p}}_1) \chi_{\frac{1}{2}}(3) \right]_I Y_{\ell_2}(\widehat{\boldsymbol{q}}_1) \right]_{JJ_z} \phi(\alpha_1)\phi(\alpha_2) \\
&\qquad\qquad \text{in } jj\text{−coupling}
\end{aligned}
\tag{2·7}
$$

The partial wave decomposition is carried out, for example, as

$$
|\boldsymbol{p}_\alpha \boldsymbol{q}_\alpha\rangle = \sum_\alpha |p_\alpha, q_\alpha; \alpha\rangle \langle \alpha | \widehat{\boldsymbol{p}_\alpha \boldsymbol{q}_\alpha}\rangle
\tag{2·8}
$$

Coordinate transformation in momentum space

$$\begin{aligned}
\begin{pmatrix} \boldsymbol{p}_1 \\ \boldsymbol{q}_1 \end{pmatrix} &= \begin{pmatrix} -\frac{\zeta}{4+\zeta} & -\frac{8+\zeta}{2(4+\zeta)} \\ 1 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \boldsymbol{p}_3 \\ \boldsymbol{q}_3 \end{pmatrix} \\
\begin{pmatrix} \boldsymbol{p}_2 \\ \boldsymbol{q}_2 \end{pmatrix} &= \begin{pmatrix} -\frac{4}{4+\zeta} & -\frac{\zeta(8+\zeta)}{(4+\zeta)^2} \\ 1 & -\frac{4}{4+\zeta} \end{pmatrix} \begin{pmatrix} \boldsymbol{p}_1 \\ \boldsymbol{q}_1 \end{pmatrix} \\
&= \begin{pmatrix} -\frac{\zeta}{4+\zeta} & \frac{8+\zeta}{2(4+\zeta)} \\ -1 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \boldsymbol{p}_3 \\ \boldsymbol{q}_3 \end{pmatrix}
\end{aligned} \tag{2·9}$$

Faddeev equation

$$\begin{aligned}
&\Psi(\boldsymbol{p}_3, \boldsymbol{q}_3) = u(\boldsymbol{p}_3, \boldsymbol{q}_3) + v(-\boldsymbol{p}_1, \boldsymbol{q}_1) + v(\boldsymbol{p}_2, \boldsymbol{q}_2) \\
&\qquad\qquad \text{with} \qquad u(-\boldsymbol{p}_3, \boldsymbol{q}_3) = u(\boldsymbol{p}_3, \boldsymbol{q}_3)
\end{aligned} \tag{2·10}$$

The partial wave decomposition of the Faddeev equation is given by

$$\begin{aligned}
u_\gamma(p,q) &= \left[E - \frac{\hbar^2}{4M_N}\left(p^2 + \frac{8+\zeta}{4\zeta}q^2\right)\right]^{-1} \int_0^\infty q'^2 dq' \int_{-1}^1 dx \\
&\times \langle p|\widetilde{T}_\lambda\left(E - \frac{\hbar^2}{4M_N}\frac{8+\zeta}{4\zeta}q^2, \varepsilon_\gamma\right)|p_1\rangle \sum_\beta \frac{1}{{p_1}^\lambda} g_{\gamma\beta}(q,q';x) \frac{1}{{p_2}^{\ell_1}} v_\beta(p_2,q') \\
v_\beta(p,q) &= \left[E - \frac{\hbar^2}{8M_N}\left(\frac{4+\zeta}{\zeta}p^2 + \frac{8+\zeta}{4+\zeta}q^2\right)\right]^{-1} \frac{1}{2}\int_0^\infty q'^2 dq' \int_{-1}^1 dx \\
&\times \left\{ \langle p|T_{\ell_1}\left(E - \frac{\hbar^2}{8M_N}\frac{8+\zeta}{4+\zeta}q^2, \varepsilon_\beta\right)|{p_1}'\rangle \sum_\gamma \frac{1}{{p_1'}^{\ell_1}} g_{\beta\gamma}(q,q';x) \frac{1}{{p_2'}^\lambda} u_\gamma(p_2',q') \right. \\
&\left. + \langle p|T_{\ell_1}\left(E - \frac{\hbar^2}{8M_N}\frac{8+\zeta}{4+\zeta}q^2, \varepsilon_\beta\right)|\widetilde{p_1}\rangle \sum_{\beta'} \frac{1}{\widetilde{p_1}^{\ell_1}} g_{\beta\beta'}(q,q';x) \frac{1}{\widetilde{p_2}^{\ell_1'}} v_{\beta'}(\widetilde{p_2},q') \right\}
\end{aligned} \tag{2·11}$$

where

$$\left\{ \begin{array}{c} p_1 = p\left(q', \frac{1}{2}q; x\right) \\ p_2 = p\left(q, \frac{\zeta}{4+\zeta}q'; x\right) \end{array} \right. \qquad \left\{ \begin{array}{c} p_1' = p\left(q', \frac{\zeta}{4+\zeta}q; x\right) \\ p_2' = p\left(q, \frac{1}{2}q'; x\right) \end{array} \right. \qquad \left\{ \begin{array}{c} \widetilde{p}_1 = p\left(q', \frac{4}{4+\zeta}q; x\right) \\ \widetilde{p}_2 = p\left(q, \frac{4}{4+\zeta}q'; x\right) \end{array} \right. \tag{2·12}$$

with $p(q,q';x) = \sqrt{q^2 + q'^2 + 2qq'x}$. The rearrangement factors are given by

$$\begin{aligned}
&g_{\gamma\beta}(q,q';x) = g_{\beta\gamma}(q',q;x) \\
&= \sum_{\lambda_1+\lambda_2=\lambda} \sum_{\lambda_1'+\lambda_2'=\ell_1} q^{\lambda_1'+\lambda_2} q'^{\lambda_1+\lambda_2'} \left(\frac{1}{2}\right)^{\lambda_2} \left(\frac{\zeta}{4+\zeta}\right)^{\lambda_2'} \sum_k (2k+1)\ g_{\gamma\beta}^{\lambda_1\lambda_1'k}\ P_k(x) \\
&g_{\gamma\beta}^{\lambda_1\lambda_1'k} = G_{(\lambda\ell),(\ell_1\ell_2)}^{\lambda_1\lambda_1'kL} \qquad\qquad \text{in } LS\text{−coupling} \\
&= \sum_L (-1)^{I+J+L+\ell_1+1} \widehat{j}\widehat{I} \left\{ \begin{array}{ccc} j & \ell & \frac{1}{2} \\ L & J & \lambda \end{array} \right\} \left(\widehat{L}\right)^2 \left\{ \begin{array}{ccc} J & L & \frac{1}{2} \\ \ell_1 & I & \ell_2 \end{array} \right\} G_{(\lambda\ell),(\ell_1\ell_2)}^{\lambda_1\lambda_1'kL} \\
&\qquad\qquad \text{in } jj\text{−coupling}
\end{aligned} \tag{2·13}$$

and

$$
\begin{aligned}
& g_{\beta\beta'}(q,q';x) \\
&= \sum_{\lambda_1+\lambda_2=\ell_1} \sum_{\lambda_1'+\lambda_2'=\ell_1'} q^{\lambda_1'+\lambda_2} q'^{\lambda_1+\lambda_2'} \left(\frac{4}{4+\zeta}\right)^{\lambda_2+\lambda_2'} \sum_k (2k+1)\ g_{\beta\beta'}^{\lambda_1\lambda_1'k}\ P_k(x) \\
& g_{\beta\beta'}^{\lambda_1\lambda_1'k} = (-1)^{\ell_1+\ell_1'} G_{(\ell_1\ell_2),(\ell_1'\ell_2')}^{\lambda_1\lambda_1'kL} \qquad \text{in } LS\text{−coupling} \\
&= \sum_L (-1)^{I-I'} \widehat{I}\widehat{I'} \left(\widehat{L}\right)^2 \left\{ \begin{array}{ccc} J & L & \frac{1}{2} \\ \ell_1 & I & \ell_2 \end{array} \right\} \left\{ \begin{array}{ccc} J & L & \frac{1}{2} \\ \ell_1' & I' & \ell_2' \end{array} \right\} G_{(\ell_1\ell_2),(\ell_1'\ell_2')}^{\lambda_1\lambda_1'kL} \\
& \qquad\qquad \text{in } jj\text{−coupling}
\end{aligned}
\tag{2·14}
$$

(Supplement)

The expressions for the rearrangement transformation are most easily obtained from the formula in § 1.4:

[u - v type]

$$
\begin{aligned}
& \langle p,q,\gamma|[1+(12)](123)|p',q',\beta\rangle = \int_{-1}^{1} dx \frac{1}{p^{\lambda+2}} \delta\left(p - p(q', \frac{1}{2}q, x)\right) \\
& \times \frac{1}{p'^{\ell_1+2}} \delta\left(p' - p(q, \frac{\zeta}{4+\zeta}q', x)\right) g_{\gamma\beta}(q,q';x)
\end{aligned}
\tag{2·15}
$$

[v - u type]

$$
\begin{aligned}
& \langle p,q,\beta|(123)^2|p',q',\gamma\rangle = \frac{1}{2}\int_{-1}^{1} dx \frac{1}{p^{\ell_1+2}} \delta\left(p - p(q', \frac{\zeta}{4+\zeta}q, x)\right) \\
& \times \frac{1}{p'^{\lambda+2}} \delta\left(p' - p(q, \frac{1}{2}q', x)\right) g_{\beta\gamma}(q,q';x)
\end{aligned}
\tag{2·16}
$$

[v - v type]

$$
\begin{aligned}
& \langle p,q,\beta|(13)|p',q',\beta'\rangle = \frac{1}{2}\int_{-1}^{1} dx \frac{1}{p^{\ell_1+2}} \delta\left(p - p(q', \frac{4}{4+\zeta}q, x)\right) \\
& \times \frac{1}{p'^{\ell_1'+2}} \delta\left(p' - p(q, \frac{4}{4+\zeta}q', x)\right) g_{\beta\beta'}(q,q';x)
\end{aligned}
\tag{2·17}
$$

Calculation of ε_γ and ε_β

The rearrangement coefficients for the normalized Jacobi coordinates in the momentum representation yield

$$
(m_\alpha + m_\gamma)\widehat{\boldsymbol{p}}_\beta^2 + (m_\beta + m_\gamma)\widehat{\boldsymbol{p}}_\alpha^2 = m_\gamma \widehat{\boldsymbol{p}}_\gamma^2 + M\widehat{\boldsymbol{q}}_\gamma^2 \tag{2·18}
$$

where $M = m_\alpha + m_\beta + m_\gamma$. We multiply $\hbar^2/2$ and write this in the symmetric form

$$(m_\beta + m_\gamma)h_\alpha + (m_\gamma + m_\alpha)h_\beta + (m_\alpha + m_\beta)h_\gamma = MH_0 \tag{2·19}$$

For two identical particles with $m_\alpha = m_\beta$, this relationship yields

$$h_\alpha + h_\beta = \frac{M}{m_\beta + m_\gamma}H_0 - \frac{2m_\beta}{m_\beta + m_\gamma}h_\gamma \tag{2·20}$$

and

$$\langle\Psi|h_\beta|\Psi\rangle = \frac{M}{2(m_\beta + m_\gamma)}\langle\Psi|H_0|\Psi\rangle - \frac{m_\beta}{m_\beta + m_\gamma}\langle\Psi|h_\gamma|\Psi\rangle \tag{2·21}$$

This implies that $\langle\Psi|h_\beta|\Psi\rangle$ is easily obtained if $\langle\Psi|h_\gamma|\Psi\rangle$ is obtained. We further use $\langle\Psi|V_\gamma|\Psi\rangle = \langle\psi_\gamma|E - H_0|\Psi\rangle$ etc. and find

$$\begin{aligned}
\varepsilon_\gamma &= E\langle\psi_\gamma|\Psi\rangle - \langle\psi_\gamma|H_0|\Psi\rangle + \langle\Psi|h_\gamma|\Psi\rangle \\
\varepsilon_\beta &= E\langle\psi_\beta|\Psi\rangle + \frac{M}{2(m_\beta + m_\gamma)}\langle\psi_\gamma|H_0|\Psi\rangle + \frac{m_\beta}{m_\beta + m_\gamma}\left[\langle\psi_\beta|H_0|\Psi\rangle - \langle\Psi|h_\gamma|\Psi\rangle\right]
\end{aligned} \tag{2·22}$$

For $\alpha\alpha\Lambda$ system, the second expression is

$$\varepsilon_\beta = E\langle\psi_\beta|\Psi\rangle + \frac{8+\zeta}{2(4+\zeta)}\langle\psi_\gamma|H_0|\Psi\rangle + \frac{4}{4+\zeta}\left[\langle\psi_\beta|H_0|\Psi\rangle - \langle\Psi|h_\gamma|\Psi\rangle\right] \tag{2·23}$$

We need to calculate the overlap and

$$\begin{aligned}
\langle\psi_\gamma|H_0|\Psi\rangle &= \langle\psi_\gamma|H_0|\psi_\gamma\rangle + 2\ \langle\psi_\gamma|H_0|\psi_\beta\rangle \\
\langle\psi_\beta|H_0|\Psi\rangle &= \langle\psi_\gamma|H_0|\psi_\beta\rangle + \langle\psi_\beta|H_0|\psi_\beta + \psi_\alpha\rangle \\
\langle\Psi|h_\gamma|\Psi\rangle &= \langle\psi_\gamma|h_\gamma|\psi_\gamma\rangle + 4\ \langle\psi_\gamma|h_\gamma|\psi_\beta\rangle + 2\ \langle\psi_\beta|h_\gamma|\psi_\beta + \psi_\alpha\rangle
\end{aligned} \tag{2·24}$$

If we use u and v for the Faddeev components, we obtain

$$\begin{aligned}
\langle\psi_\gamma|H_0|\Psi\rangle &= \langle u|H_0|u\rangle + 2\ \langle u|H_0|{P_{(123)}}^2 v\rangle \\
\langle\psi_\beta|H_0|\Psi\rangle &= \langle u|H_0|{P_{(123)}}^2 v\rangle + \langle v|H_0|v \pm P_{(23)}v\rangle \\
\langle\Psi|h_\gamma|\Psi\rangle &= \langle u|h|u\rangle + 4\ \langle u|h|{P_{(123)}}^2 v\rangle + 2\ \langle v|P_{(123)}hP_{(123)}{}^{-1}|v \pm P_{(23)}v\rangle
\end{aligned} \tag{2·25}$$

where $h = h_\gamma$. The explicit expressions are given by

$$\begin{aligned}
\langle\psi_\gamma|H_0|\Psi\rangle &= \sum_\gamma \int_0^\infty p^2 dp q^2 dq \frac{\hbar^2}{4M_N}\left(p^2 + \frac{8+\zeta}{4\zeta}q^2\right)\left(u_\gamma(p,q)\right)^2 \\
&+ \sum_{\gamma,\beta}\int_0^\infty q^2 dq {q'}^2 dq' \int_{-1}^1 dx u_\gamma(p_1, q)\frac{\hbar^2}{4M_N}\left({p_1}^2 + \frac{8+\zeta}{4\zeta}q^2\right)
\end{aligned}$$

$$\times\frac{1}{{p_1}^{\lambda}}g_{\gamma\beta}(q,q';x)\frac{1}{{p_2}^{\ell_1}}v_\beta(p_2,q')$$

$$\langle\psi_\beta|H_0|\Psi\rangle=\sum_\beta\int_0^\infty p^2dpq^2dq\frac{\hbar^2}{8M_N}\left(\frac{4+\zeta}{\zeta}p^2+\frac{8+\zeta}{4+\zeta}q^2\right)\left(v_\beta(p,q)\right)^2$$

$$+\sum_{\beta,\beta'}\int_0^\infty q^2dqq'^2dq'\int_{-1}^1 dxv_\beta(\widetilde{p_1},q)\frac{\hbar^2}{2\cdot 8M_N}\left(\frac{4+\zeta}{\zeta}\widetilde{p_1}^2+\frac{8+\zeta}{4+\zeta}q^2\right)$$

$$\times\frac{1}{\widetilde{p_1}^{\ell_1}}g_{\beta\beta'}(q,q';x)\frac{1}{\widetilde{p_2}^{\ell_1'}}v_{\beta'}(\widetilde{p_2},q')$$

$$+\frac{1}{2}\sum_{\gamma,\beta}\int_0^\infty q^2dqq'^2dq'\int_{-1}^1 dxu_\gamma(p_1,q)\frac{\hbar^2}{4M_N}\left({p_1}^2+\frac{8+\zeta}{4\zeta}q^2\right)$$

$$\times\frac{1}{{p_1}^{\lambda}}g_{\gamma\beta}(q,q';x)\frac{1}{{p_2}^{\ell_1}}v_\beta(p_2,q') \qquad (2\cdot 26)$$

The overlapping integrals are obtained by setting $H_0\to 1$. Finally, $\langle\Psi|h_\gamma|\Psi\rangle$ is given by

$$\langle\Psi|h_\gamma|\Psi\rangle=\sum_\gamma\int_0^\infty p^2dpq^2dq\frac{\hbar^2}{4M_N}p^2\left(u_\gamma(p,q)\right)^2$$

$$+\sum_{\gamma,\beta}\int_0^\infty q^2dqq'^2dq'\int_{-1}^1 dxu_\gamma(p_1,q)\frac{\hbar^2}{2M_N}{p_1}^2\frac{1}{{p_1}^{\lambda}}g_{\gamma\beta}(q,q';x)\frac{1}{{p_2}^{\ell_1}}v_\beta(p_2,q')$$

$$+\sum_{\beta,\beta'}\int_0^\infty p^2dpq^2dqv_\beta(p,q)\frac{\hbar^2}{8M_N}\left\{\left[p^2+\left(\frac{8+\zeta}{4+\zeta}\right)^2q^2\right]\delta_{\beta,\beta'}+\frac{2(8+\zeta)}{4+\zeta}pqf_{\beta\beta'}\right\}$$

$$\times v_{\beta'}(p,q)$$

$$+\sum_{\beta,\beta'}\int_0^\infty q^2dqq'^2dq'\int_{-1}^1 dxv_\beta(\widetilde{p_1},q)\frac{\hbar^2}{16M_N}\left(q^2+q'^2-2qq'x\right)\frac{1}{\widetilde{p_1}^{\ell_1}}$$

$$\times g_{\beta\beta'}(q,q';x)\frac{1}{\widetilde{p_2}^{\ell_1'}}v_{\beta'}(\widetilde{p_2},q') \qquad (2\cdot 27)$$

Here, $f_{\beta\beta'}$ is given by

$$f_{\beta\beta'}=\int d\widehat{\boldsymbol{p}}_2d\widehat{\boldsymbol{q}}_2\langle\beta|\widehat{\boldsymbol{p}}_2\widehat{\boldsymbol{q}}_2\rangle(\widehat{\boldsymbol{p}}_2\cdot\widehat{\boldsymbol{q}}_2)\langle\widehat{\boldsymbol{p}}_2\widehat{\boldsymbol{q}}_2|\beta'\rangle$$

$$=(-1)^{\ell_1+\ell_2'+L}\widehat{\ell_1}\widehat{\ell_2}\langle\ell_1010|\ell_1'\rangle\langle\ell_2010|\ell_2'\rangle\left\{\begin{array}{ccc}\ell_1 & \ell_2 & L\\ \ell_2' & \ell_1' & 1\end{array}\right\}\qquad LS-\text{coupling}$$

$$=(-1)^{I-I'+\ell_1+\ell_2'}\widehat{\ell_1}\widehat{\ell_2}\widehat{I}\widehat{I'}\langle\ell_1010|\ell_1'\rangle\langle\ell_2010|\ell_2'\rangle$$

$$\times\sum_L(-1)^L(\widehat{L})^2\left\{\begin{array}{ccc}J & L & \frac{1}{2}\\ \ell_1 & I & \ell_2\end{array}\right\}\left\{\begin{array}{ccc}J & L & \frac{1}{2}\\ \ell_1' & I' & \ell_2'\end{array}\right\}\left\{\begin{array}{ccc}\ell_1 & \ell_2 & L\\ \ell_2' & \ell_1' & 1\end{array}\right\}$$

$$jj-\text{coupling} \qquad (2\cdot 28)$$

$\boldsymbol{\Lambda\alpha}$ and $\boldsymbol{\alpha\alpha}$ $\boldsymbol{T}$-matrix

The $\Lambda\alpha$ T-matrix is obtained by solving the partial-wave Lippmann-Schwinger equation

$$T_\ell(p,p';E) = V_\ell(p,p') - \frac{4\pi}{(2\pi)^3}\frac{2\mu}{\hbar^2}\int_0^\infty k^2 dk V_\ell(p,k)\frac{1}{\gamma^2+k^2}T_\ell(k,p';E) \qquad (2\cdot29)$$

where $\mu = (4\zeta/(4+\zeta))M_N$ is the $\Lambda\alpha$ reduced mass and $E = -(\hbar^2/2\mu)\gamma^2$ is the negative energy. $V_\ell(p,p')$ is the partial wave component for the $\Lambda\alpha$ Born amplitude $V(\boldsymbol{p},\boldsymbol{p}')$:

$$\langle \boldsymbol{p}|V|\boldsymbol{p}'\rangle = \frac{1}{(2\pi)^3}V(\boldsymbol{p},\boldsymbol{p}') = \frac{4\pi}{(2\pi)^3}\sum_\ell V_\ell(p,p')\sum_m Y_{\ell m}(\widehat{\boldsymbol{p}})^* Y_{\ell m}(\widehat{\boldsymbol{p}'}) \qquad (2\cdot30)$$

The Born amplitude is obtained from the folding procedure with respect to the ΛN effective interaction through the standard procedure.

$$\begin{aligned} V(\boldsymbol{q}_f,\boldsymbol{q}_i) &= \langle e^{i\boldsymbol{q}_f\boldsymbol{r}}|V|e^{i\boldsymbol{q}_i\boldsymbol{r}}\rangle = \langle e^{i\boldsymbol{q}_f\boldsymbol{r}}\xi_\Lambda\phi_\alpha|\sum_{i=1}^4 v_{i5}|e^{i\boldsymbol{q}_i\boldsymbol{r}}\xi_\Lambda\phi_\alpha\rangle \\ &= \langle\delta(\boldsymbol{X}_G)e^{i\boldsymbol{q}_f\boldsymbol{r}}\xi_\Lambda\phi_\alpha|\sum_{i=1}^4 v_{i5}|1\cdot e^{i\boldsymbol{q}_i\boldsymbol{r}}\xi_\Lambda\phi_\alpha\rangle \\ &= \left(\frac{\gamma}{2\pi}\right)^{\frac{3}{2}} e^{\frac{1}{4\gamma}({\boldsymbol{q}_f}^2+{\boldsymbol{q}_i}^2)}\int d\boldsymbol{a}d\boldsymbol{b}e^{-i\boldsymbol{q}_f\cdot\boldsymbol{a}+i\boldsymbol{q}_i\cdot\boldsymbol{b}}G(\boldsymbol{a},\boldsymbol{b}) \end{aligned} \qquad (2\cdot31)$$

with

$$\begin{aligned} G(\boldsymbol{a},\boldsymbol{b}) &= \left(\frac{\gamma_G}{2\pi}\right)^{\frac{3}{2}}\int d\boldsymbol{R}\; G(\boldsymbol{a},\boldsymbol{b};\boldsymbol{R}) \\ G(\boldsymbol{a},\boldsymbol{b};\boldsymbol{R}) &= \langle\psi_\Lambda(\boldsymbol{a})\psi_\alpha|\sum_{i=1}^4 v_{i5}|\psi_\Lambda(\boldsymbol{R}+\boldsymbol{b})\psi_\alpha(\boldsymbol{R})\rangle \end{aligned} \qquad (2\cdot32)$$

For the spatial part u in $v_{i5} = u(\boldsymbol{x}_i-\boldsymbol{x}_5)\omega_{i5}$, we can derive a convenient formula

$$V(\boldsymbol{q}_f,\boldsymbol{q}_i) = e^{-\frac{3}{8\nu}\frac{\boldsymbol{k}^2}{4}}\left(\frac{2}{3\pi\nu}\right)^{\frac{3}{2}}\int d\boldsymbol{P}e^{-\frac{2}{3\nu}\left(\boldsymbol{P}-\frac{3}{4}\boldsymbol{q}\right)^2}u\left(\boldsymbol{k},\boldsymbol{q}-\frac{\zeta}{\zeta+1}\boldsymbol{P};\boldsymbol{P}\right) \qquad (2\cdot33)$$

where

$$\boldsymbol{k} = \boldsymbol{q}_f - \boldsymbol{q}_i \qquad , \qquad \boldsymbol{q} = \frac{1}{2}\left(\boldsymbol{q}_f+\boldsymbol{q}_i\right) \qquad (2\cdot34)$$

and $u(\boldsymbol{k},\boldsymbol{q};\boldsymbol{P})$ is defined through

$$\langle \boldsymbol{p}_1\boldsymbol{p}_2|u|\boldsymbol{p}_1{}'\boldsymbol{p}_2{}'\rangle = \frac{1}{(2\pi)^3}\delta(\boldsymbol{P}-\boldsymbol{P}')u(\boldsymbol{k}',\boldsymbol{q}';\boldsymbol{P}) \qquad (2\cdot35)$$

with

$$\left\{\begin{array}{c} \boldsymbol{p} = \frac{1}{\zeta+1}(\boldsymbol{p}_1-\zeta\boldsymbol{p}_2) \\ \boldsymbol{P} = \boldsymbol{p}_1+\boldsymbol{p}_2 \end{array}\right. \qquad \left\{\begin{array}{ll} \boldsymbol{p}_1 = \frac{\zeta}{1+\zeta}\boldsymbol{P}+\boldsymbol{p} & \text{for } \Lambda \\ \boldsymbol{p}_2 = \frac{1}{1+\zeta}\boldsymbol{P}-\boldsymbol{p} & \text{for } N \end{array}\right. \qquad (2\cdot36)$$

(also for $\boldsymbol{p}_1{}'$ and $\boldsymbol{p}_2{}'$) and

$$\left\{ \begin{array}{c} \boldsymbol{k}' = \boldsymbol{p} - \boldsymbol{p}' \\ \boldsymbol{q}' = \frac{1}{2}(\boldsymbol{p} + \boldsymbol{p}') \end{array} \right. \qquad \left\{ \begin{array}{c} \boldsymbol{p} = \boldsymbol{q}' + \frac{1}{2}\boldsymbol{k}' \\ \boldsymbol{p}' = \boldsymbol{q}' - \frac{1}{2}\boldsymbol{k}' \end{array} \right. \tag{2·37}$$

For a simple local Gaussian interaction with $u(r) = e^{-\kappa r^2}$ and $u(r)P_r$, we have

$$\begin{aligned} u(\boldsymbol{k}, \boldsymbol{q}) &= \left(\frac{\pi}{\kappa}\right)^{\frac{3}{2}} \exp\left\{-\frac{\boldsymbol{k}^2}{4\kappa}\right\} \qquad \text{for} \qquad u(r) = e^{-\kappa r^2} \\ u(\boldsymbol{k}, \boldsymbol{q}; \boldsymbol{P}) &= \left(\frac{\pi}{\kappa}\right)^{\frac{3}{2}} \exp\left\{-\frac{1}{\kappa}\left(\boldsymbol{q} + \frac{1}{2}\frac{\zeta - 1}{\zeta + 1}\boldsymbol{P}\right)^2\right\} \\ &\qquad \text{for} \qquad u(r) = e^{-\kappa r^2} P_r \end{aligned} \tag{2·38}$$

Then the integral is carried out easily and we obtain

$$\begin{aligned} V_d(\boldsymbol{q}_f, \boldsymbol{q}_i) &= \left(\frac{\pi}{\kappa}\right)^{\frac{3}{2}} \exp\left\{-\frac{1}{4}\left(\frac{3}{8\nu} + \frac{1}{\kappa}\right)\boldsymbol{k}^2\right\} \qquad \text{for} \qquad u(r) = e^{-\kappa r^2} \\ V_e(\boldsymbol{q}_f, \boldsymbol{q}_i) &= \left(\frac{8\pi}{3}\frac{1}{\nu + \frac{8}{3}\kappa}\right)^{\frac{3}{2}} \exp\left\{-\frac{3}{8\nu}\frac{\boldsymbol{k}^2}{4} - \frac{25}{24}\frac{1}{\nu + \frac{8}{3}\kappa}\boldsymbol{q}^2\right\} \\ &\qquad \text{for} \qquad u(r) = e^{-\kappa r^2} P_r \end{aligned} \tag{2·39}$$

Incorporating the spin-isospin factors, the full $V(\boldsymbol{q}_f, \boldsymbol{q}_i)$ is given by

$$V(\boldsymbol{q}_f, \boldsymbol{q}_i) = X_d \; V_d(\boldsymbol{q}_f, \boldsymbol{q}_i) + X_e \; V_e(\boldsymbol{q}_f, \boldsymbol{q}_i) \tag{2·40}$$

with

$$\left\{ \begin{array}{c} X_d \\ X_e \end{array} \right\} = \langle \xi_\Lambda \chi_\alpha | \sum_{i=1}^{4} \left\{ \begin{array}{c} \omega_{i5}^d \\ \omega_{i5}^e \end{array} \right\} | \xi_\Lambda \chi_\alpha \rangle \tag{2·41}$$

The partial wave decomposition is given by

$$\begin{aligned} V_\ell^d(q_f, q_i; \kappa) &= \left(\frac{\pi}{\kappa}\right)^{\frac{3}{2}} \exp\left\{-\frac{1}{4}\left(\frac{3}{8\nu} + \frac{1}{\kappa}\right)\left({q_f}^2 + {q_i}^2\right)\right\} \; i_\ell\left(\frac{1}{2}\left(\frac{3}{8\nu} + \frac{1}{\kappa}\right) q_f q_i\right) \\ &\qquad \text{for} \qquad u(r) = e^{-\kappa r^2} \end{aligned} \tag{2·42}$$

$$\begin{aligned} V_\ell^e(q_f, q_i; \kappa) &= \left(\frac{8\pi}{3}\frac{1}{\nu + \frac{8}{3}\kappa}\right)^{\frac{3}{2}} \exp\left\{-\frac{1}{4}\left(\frac{3}{8\nu} + \frac{25}{24}\frac{1}{\nu + \frac{8}{3}\kappa}\right)\left({q_f}^2 + {q_i}^2\right)\right\} \\ &\quad \times i_\ell\left(\frac{1}{2}\left(\frac{3}{8\nu} - \frac{25}{24}\frac{1}{\nu + \frac{8}{3}\kappa}\right) q_f q_i\right) \quad \text{for} \quad u(r) = e^{-\kappa r^2} P_r \end{aligned} \tag{2·43}$$

where $i_\ell(x) = i^\ell j_\ell(-ix)$.

The Hiyama potential for the ΛN effective interaction is parametrized as

$$\begin{aligned} v_{\Lambda N} = \sum_{i=1}^{3} \Bigg[&\left(v_{0\ \text{even}}^{(i)} + v_{\sigma\sigma\ \text{even}}^{(i)}\right)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\frac{1 + P_r}{2} \\ &+ \left(v_{0\ \text{odd}}^{(i)} + v_{\sigma\sigma\ \text{odd}}^{(i)}\right)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\frac{1 - P_r}{2}\Bigg] \; e^{-\left(\frac{r}{\beta_i}\right)^2} \end{aligned} \tag{2·44}$$

Here the spin-spin term does not contribute to the spin saturated α. Thus we find

$$V_\ell(q_f,q_i)=\sum_{i=1}^{3}\left[X_d^i\ V_\ell^d\left(q_f,q_i;\frac{1}{\beta_i}\right)+X_e^i\ V_\ell^e\left(q_f,q_i;\frac{1}{\beta_i}\right)\right] \tag{2·45}$$

with

$$X_d^i=2\ \left({v_0^{(i)}}_{\rm even}+{v_0^{(i)}}_{\rm odd}\right)\qquad\text{and}\qquad X_e^i=2\ \left({v_0^{(i)}}_{\rm even}-{v_0^{(i)}}_{\rm odd}\right) \tag{2·46}$$

The explicit values for ${v_0^{(i)}}_{\rm even}$ and ${v_0^{(i)}}_{\rm odd}$ with $i=1,\ 2,\ 3$ are given in their paper, Table V of Prog. Theor. Phys. 97 (1997), 881.

Expressions for numerical calculations

(Faddeev equation)

$$\begin{aligned}
u_{\gamma i\mu}&=\sum_k(G\widetilde{T})_{\lambda ik\mu}\sum_{\nu\kappa}S^{(1)}_{k\nu\mu\kappa}\sum_\beta B_{\gamma\mu,\beta\nu,\kappa}\sum_j S^{(2)}_{j\mu\nu\kappa}\ v_{\beta j\nu}\\
v_{\beta i\mu}&=\sum_k(GT)_{\ell_1 ik\mu}\sum_{\nu\kappa}\left\{S^{(2)}_{k\nu\mu\kappa}\sum_\alpha\frac{1}{2}B_{\alpha\nu,\beta\mu,\kappa}\sum_j S^{(1)}_{j\mu\nu\kappa}\ u_{\alpha j\nu}\right.\\
&\qquad\left.+S^{(3)}_{k\nu\mu\kappa}\sum_{\beta'}B_{\beta\mu,\beta'\nu,\kappa}\sum_j S^{(3)}_{j\mu\nu\kappa}\ v_{\beta' j\nu}\right\}
\end{aligned} \tag{2·47}$$

where

$$u_{\gamma i\mu}=u_\gamma(p_i,q_\mu)q_\mu\sqrt{w_\mu}\qquad\text{and}\qquad v_{\beta i\mu}=v_\beta(p_i,q_\mu)q_\mu\sqrt{w_\mu} \tag{2·48}$$

and

$$\begin{aligned}
(G\widetilde{T})_{\lambda ik\mu}&=\left[E-\frac{\hbar^2}{4M_N}\left({p_i}^2+\frac{8+\zeta}{4\zeta}{q_\mu}^2\right)\right]^{-1}\frac{4\pi}{(2\pi)^3}\widetilde{T}_\lambda\left(p_i,p_k;E-\frac{\hbar^2}{4M_N}\frac{8+\zeta}{4\zeta}{q_\mu}^2,\varepsilon_\gamma\right)\\
(GT)_{\ell_1 ik\mu}&=\left[E-\frac{\hbar^2}{8M_N}\left(\frac{4+\zeta}{\zeta}{p_i}^2+\frac{8+\zeta}{4+\zeta}{q_\mu}^2\right)\right]^{-1}\\
&\quad\times\frac{4\pi}{(2\pi)^3}T_{\ell_1}\left(p_i,p_k;E-\frac{\hbar^2}{8M_N}\frac{8+\zeta}{4+\zeta}{q_\mu}^2,\varepsilon_\beta\right)
\end{aligned} \tag{2·49}$$

and

$$S^{(n)}_{i\mu\nu\kappa}=S_i(p^{(n)}_{\mu\nu\kappa})\qquad(n=1\sim3) \tag{2·50}$$

with

$$\begin{aligned}
p^{(1)}_{\mu\nu\kappa}&=p\left(q_\mu,\frac{1}{2}q_\nu,x_\kappa\right)\\
p^{(2)}_{\mu\nu\kappa}&=p\left(q_\mu,\frac{\zeta}{4+\zeta}q_\nu,x_\kappa\right)\\
p^{(3)}_{\mu\nu\kappa}&=p\left(q_\mu,\frac{4}{4+\zeta}q_\nu,x_\kappa\right)
\end{aligned} \tag{2·51}$$

and $p(q,q',x)=\sqrt{q^2+q'^2+2qq'x}$. The rearrangement coefficients are given by

$$B_{\gamma\mu,\beta\nu,\kappa}=\left(\frac{1}{p^{(1)}_{\nu\mu\kappa}}\right)^{\lambda} g_{\gamma\beta}(q_\mu,q_\nu;x_\kappa)\left(\frac{1}{p^{(2)}_{\mu\nu\kappa}}\right)^{\ell_1} q_\mu q_\nu\sqrt{w_\mu w_\nu}w_\kappa$$

$$B_{\beta\mu,\gamma\nu,\kappa}=\frac{1}{2}B_{\gamma\nu,\beta\mu,\kappa} \tag{2·52}$$

$$B_{\beta\mu,\beta'\nu,\kappa}=\frac{1}{2}\left(\frac{1}{p^{(3)}_{\nu\mu\kappa}}\right)^{\ell_1} g_{\beta\beta'}(q_\mu,q_\nu;x_\kappa)\left(\frac{1}{p^{(3)}_{\mu\nu\kappa}}\right)^{\ell_1{}'} q_\mu q_\nu\sqrt{w_\mu w_\nu}w_\kappa \tag{2·53}$$

where $g_{\beta\gamma}(q_\mu,q_\nu;x_\kappa)=g_{\gamma\beta}(q_\nu,q_\mu;x_\kappa)$.

(ε_γ and ε_β)

$$\langle\psi_\gamma|\begin{Bmatrix}1\\H_0\end{Bmatrix}|\Psi\rangle=\sum_{i\mu}\begin{Bmatrix}1\\ \frac{\hbar^2}{4M_N}\left({p_i}^2+\frac{8+\zeta}{4\zeta}{q_\mu}^2\right)\end{Bmatrix}\left(\sum_\gamma(u_{\gamma i\mu})^2\right){p_i}^2w_i$$
$$+\sum_{\gamma\mu}\sum_{\nu\kappa}\left(\sum_i u_{\gamma i\mu}S^{(1)}_{i\nu\mu\kappa}\right)\begin{Bmatrix}1\\ \frac{\hbar^2}{4M_N}\left({p^{(1)}_{\nu\mu\kappa}}^2+\frac{8+\zeta}{4\zeta}{q_\mu}^2\right)\end{Bmatrix}\sum_\beta B_{\gamma\mu,\beta\nu,\kappa}\sum_j S^{(2)}_{j\mu\nu\kappa}v_{\beta j\nu}$$

$$\langle\psi_\beta|\begin{Bmatrix}1\\H_0\end{Bmatrix}|\Psi\rangle=\sum_{i\mu}\begin{Bmatrix}1\\ \frac{\hbar^2}{8M_N}\left(\frac{4+\zeta}{\zeta}{p_i}^2+\frac{8+\zeta}{4+\zeta}{q_\mu}^2\right)\end{Bmatrix}\left(\sum_\beta(v_{\beta i\mu})^2\right){p_i}^2w_i$$
$$+\sum_{\beta\mu}\sum_{\nu\kappa}\left(\sum_i v_{\beta i\mu}S^{(3)}_{i\nu\mu\kappa}\right)\begin{Bmatrix}1\\ \frac{\hbar^2}{8M_N}\left(\frac{4+\zeta}{\zeta}{p^{(3)}_{\nu\mu\kappa}}^2+\frac{8+\zeta}{4+\zeta}{q_\mu}^2\right)\end{Bmatrix}\sum_{\beta'} B_{\beta\mu,\beta'\nu,\kappa}\sum_j S^{(3)}_{j\mu\nu\kappa}v_{\beta' j\nu}$$
$$+\frac{1}{2}\sum_{\gamma\mu}\sum_{\nu\kappa}\left(\sum_i u_{\gamma i\mu}S^{(1)}_{i\nu\mu\kappa}\right)\begin{Bmatrix}1\\ \frac{\hbar^2}{4M_N}\left({p^{(1)}_{\nu\mu\kappa}}^2+\frac{8+\zeta}{4\zeta}{q_\mu}^2\right)\end{Bmatrix}\sum_\beta B_{\gamma\mu,\beta\nu,\kappa}\sum_j S^{(2)}_{j\mu\nu\kappa}v_{\beta j\nu}$$

$$\langle\Psi|h_\gamma|\Psi\rangle=\frac{\hbar^2}{4M_N}\sum_{i\mu}\left(\sum_\gamma(u_{\gamma i\mu})^2\right){p_i}^4w_i$$
$$+\frac{\hbar^2}{8M_N}\sum_{i\mu}\left\{\left[{p_i}^2+\left(\frac{8+\zeta}{4+\zeta}\right)^2{q_\mu}^2\right]\left(\sum_\beta(v_{\beta i\mu})^2\right)+\frac{2(8+\zeta)}{4+\zeta}p_iq_\mu\left(\sum_{\beta\beta'}v_{\beta i\mu}f_{\beta\beta'}v_{\beta' i\mu}\right)\right\}{p_i}^2w_i$$
$$+\frac{\hbar^2}{2M_N}\sum_{\gamma\mu}\sum_{\nu\kappa}\left(\sum_i u_{\gamma i\mu}S^{(1)}_{i\nu\mu\kappa}\right){p^{(1)}_{\nu\mu\kappa}}^2\sum_\beta B_{\gamma\mu,\beta\nu,\kappa}\sum_j S^{(2)}_{j\mu\nu\kappa}v_{\beta j\nu}$$
$$+\frac{\hbar^2}{8M_N}\sum_{\beta\mu}\sum_{\nu\kappa}\left(\sum_i v_{\beta i\mu}S^{(3)}_{i\nu\mu\kappa}\right)\left({q_\mu}^2+{q_\nu}^2-2q_\mu q_\nu x_\kappa\right)\sum_{\beta'}B_{\beta\mu,\beta'\nu,\kappa}\sum_j S^{(3)}_{j\mu\nu\kappa}v_{\beta' j\nu} \tag{2·54}$$

ε_γ and ε_β are obtained from

$$\varepsilon_\gamma=E\langle\psi_\gamma|\Psi\rangle-\langle\psi_\gamma|H_0|\Psi\rangle+\langle\Psi|h_\gamma|\Psi\rangle$$
$$\varepsilon_\beta=E\langle\psi_\beta|\Psi\rangle+\frac{8+\zeta}{2(4+\zeta)}\langle\psi_\gamma|H_0|\Psi\rangle+\frac{4}{4+\zeta}\left[\langle\psi_\beta|H_0|\Psi\rangle-\langle\Psi|h_\gamma|\Psi\rangle\right] \tag{2·55}$$

The total kinetic energy and the potential energy are obtained from

$$\langle\Psi|H_0|\Psi\rangle = \langle\psi_\gamma|H_0|\Psi\rangle + 2\ \langle\psi_\beta|H_0|\Psi\rangle$$
$$\langle\Psi|V|\Psi\rangle = E - \langle\Psi|H_0|\Psi\rangle \tag{2·56}$$

Solution of the bound state for 2α system

The 2α RGM equation

$$(\omega - H_0)\psi = V_{\rm RGM}(\omega)\psi \tag{2·57}$$

involves trivial solutions. We assume only one trivial solution $\psi = u$, and present a practical method to solve Eq. (2·57) for the bound state. Using the notation of our previous paper "Three cluster Equations using the two cluster RGM kernel" (Prog. Theor. Phys. 107 (2002) 745), we only need to solve

$$(\omega - H_0)\psi = \mathcal{V}(\varepsilon,\omega)\psi \tag{2·58}$$

where

$$\mathcal{V}(\varepsilon,\omega) = V(\varepsilon) + v(\omega)$$
$$V(\varepsilon) = (\omega - H_0) - \Lambda(\omega - H_0)\Lambda$$
$$v(\omega) = \Lambda V_{\rm RGM}(\omega)\Lambda = \Lambda(V_{\rm D} + G + \omega K)\Lambda \tag{2·59}$$

and $\Lambda = |u\rangle\langle u|$. Note that

$$\mathcal{V}(\varepsilon,\omega) = V_{\rm RGM}(\omega) - (\omega - \varepsilon)|u\rangle\langle u| \tag{2·60}$$

and $\mathcal{V}(\omega,\omega) = V_{\rm RGM}(\omega)$. Then, for an appropriate ε, the solution of Eq. (2·58) is the redundant-free solution of Eq. (2·57): $\langle u|\psi\rangle = 0$ if $\varepsilon \neq \omega$. ε can be arbitrary as long as $\varepsilon \neq \omega$, and we can choose $\varepsilon = 0$ (Saito's choice). In the momentum representation, it is the easiest to solve the eigen-value problem

$$\lambda(\omega)\ \psi = G_0(\omega)\,[\,V_{\rm RGM}(\omega) - \omega|u\rangle\langle u|\,]\ \psi \qquad \text{with} \qquad \lambda(\omega) = 1 \tag{2·61}$$

Setting $\omega = -(\hbar/2\mu)\gamma^2$ and $(1/(2\pi)^{3/2})u_n(q) = R_{n0}(q, 1/4\gamma)$ with $n = 0, 1$, we find for the 2α S-wave bound state

$$\psi(q) = \frac{1}{\sqrt{\gamma^2+q^2}} f(q)$$
$$\lambda(\omega)\ f(p) = -\int_0^\infty q^2 dq \frac{1}{\sqrt{(\gamma^2+p^2)(\gamma^2+q^2)}} \left[\ \frac{4M_N}{\hbar^2}\frac{4\pi}{(2\pi)^3} V_{\rm RGM}\left(p, q, -\frac{\hbar^2}{4M_N}\gamma^2\right)\right.$$
$$\left. +\gamma^2 \sum_{n=0}^{1} R_{n0}\left(p; \frac{1}{8\nu}\right) R_{n0}\left(q; \frac{1}{8\nu}\right)\right] f(q) \tag{2·62}$$

In the discretized form, it is given by

$$\psi_i = \psi(p_i) p_i \sqrt{w_i} = \frac{1}{\sqrt{\gamma^2 + {p_i}^2}} f(p_i) \sqrt{w_i}$$
$$\lambda\ f_i = \sum_j A_{ij} f_j$$
$$A_{ij} = -\frac{p_i p_j \sqrt{w_i w_j}}{\sqrt{(\gamma^2 + {p_i}^2)(\gamma^2 + {p_j}^2)}} \left[\frac{4M_N}{\hbar^2} \frac{4\pi}{(2\pi)^3} V_{\rm RGM}\left(p_i, p_j, -\frac{\hbar^2}{4M_N}\gamma^2\right) \right.$$
$$\left. +\gamma^2 \sum_{n=0}^{1} R_{n0}\left(p_i; \frac{1}{8\nu}\right) R_{n0}\left(p_j; \frac{1}{8\nu}\right) \right] \qquad (2\cdot63)$$

§3. ${}^{\mathbf{6}}_{\Lambda\Lambda}$He system

The Faddeev formulation of the $2\Lambda\alpha$ system is very close to the the $2\alpha\Lambda$ system, except for the entirely different mass ratio due to the exchanged role of α and Λ. We interchange $4M_N$ and M_Λ, and replace

$$\frac{1}{4}\zeta = M_\Lambda 4M_N \to 4M_N 4M_\Lambda = \frac{4}{\zeta}\ , \qquad (3\cdot1)$$

so that the replacement rule is $\zeta \to 16/\zeta$. If we restrict to the formulation of the ${}^{\mathbf{6}}_{\Lambda\Lambda}$He ground state with the total angular-momentum $J = 0$ and the isospin $T = 0$, the channel specification scheme is very simple. These are specified by

(γ channel) $\gamma = 3$ with $(\gamma\alpha\beta) = (312)$

$$\boldsymbol{p}_3 = \frac{1}{2}(\boldsymbol{k}_1 - \boldsymbol{k}_2)$$
$$\boldsymbol{q}_3 = \frac{1}{2+\zeta}\left[\zeta \boldsymbol{k}_3 - 2(\boldsymbol{k}_1 + \boldsymbol{k}_2)\right] \qquad (3\cdot2)$$

where $\zeta = M_\Lambda/M_N$. The channel wave functions are given by

$$\langle \widehat{\boldsymbol{p}}_3 \widehat{\boldsymbol{q}}_3; 123|\gamma\rangle = \begin{cases} \left[Y_{(\lambda\lambda)S}(\widehat{\boldsymbol{p}}_3 \widehat{\boldsymbol{q}}_3)\chi_S(12)\right]_0 \phi(\alpha) & (LS) \\ \left[\mathcal{Y}_{(\lambda S)\lambda}(\widehat{\boldsymbol{p}}_3; 12) Y_\lambda(\widehat{\boldsymbol{q}}_3)\right]_0 \phi(\alpha) & (jj) \end{cases} . \qquad (3\cdot3)$$

Here, $\chi_S(12) = [\chi_{1/2}(1)\chi_{1/2}(2)]_S$ is the two-particle spin function of Λ and $\mathcal{Y}_{(\lambda S)\lambda\lambda_z}(\widehat{\boldsymbol{p}}_3; 12) = [Y_\lambda(\widehat{\boldsymbol{p}}_3)\chi_S(12)]_{\lambda\lambda_z}$ is the standard space-spin wave function of the two-Λ system. We also use γ to specify a set of quantum numbers; i.e., $\gamma = [(\lambda\lambda)S, S]0$ (in LS) or $\gamma = [(\lambda S)\lambda, \lambda]0$ (in jj).

(β channel) $\beta = 2$ with $(\beta\gamma\alpha) = (231)$

$$\boldsymbol{p}_2 = \frac{1}{4+\zeta}(\zeta \boldsymbol{k}_3 - 4\boldsymbol{k}_1)$$
$$\boldsymbol{q}_2 = \frac{1}{2(2+\zeta)}\left[(4+\zeta)\boldsymbol{k}_2 - \zeta(\boldsymbol{k}_3 + \boldsymbol{k}_1)\right] \qquad (3\cdot4)$$

$$\langle \widehat{\boldsymbol{p}}_2\widehat{\boldsymbol{q}}_2;312|\beta\rangle = \begin{cases} \left[Y_{(\ell_1\ell_1)S}(\widehat{\boldsymbol{p}}_2\widehat{\boldsymbol{q}}_2)\chi_S(12)\right]_0\phi(\alpha) & (LS) \\ \left[\mathcal{Y}_{(\ell_1\frac{1}{2})I}(\widehat{\boldsymbol{p}}_2;1)\mathcal{Y}_{(\ell_1\frac{1}{2})I}(\widehat{\boldsymbol{q}}_2;2)\right]_0\phi(\alpha) & (jj) \end{cases} . \tag{3·5}$$

Here, $\mathcal{Y}_{(\ell_1 1/2)II_z}(\widehat{\boldsymbol{p}}_2;1) = [Y_{\ell_1}(\widehat{\boldsymbol{p}}_2)\chi_{1/2}(1)]_{II_z}$ etc. and $\beta = [(\ell_1\ell_1)S,S]0$ (in LS) or $\beta = [(\ell_1 1/2)I,(\ell_1 1/2)I]0$ (in jj).

(α channel) $\alpha = 1$ with $(\alpha\beta\gamma) = (123)$

$$\begin{aligned} \boldsymbol{p}_1 &= \frac{1}{4+\zeta}(4\boldsymbol{k}_2 - \zeta\boldsymbol{k}_3) \\ \boldsymbol{q}_1 &= \frac{1}{2(2+\zeta)}\left[(4+\zeta)\boldsymbol{k}_1 - \zeta(\boldsymbol{k}_2+\boldsymbol{k}_3)\right] \end{aligned} \tag{3·6}$$

with $\alpha = [(\ell_1\ell_1)S,S]0$ (in LS) or $\alpha = [(\ell_1 1/2)I,(\ell_1 1/2)I]0$ (in jj).

Note that in the γ channel the exchange of the two Λ's leads to the antisymmetry, $(-)^{\lambda+1-S} = -1$, which implies $(-)^{\lambda+S} = 1$. Therefore, the possible two-λ states are ${}^1\lambda_\lambda$ (λ=even) for $S=0$ and ${}^3\lambda_\lambda$ (λ=odd) for $S=1$. If we neglect non-central forces, the spin vales S (and also the orbital quantum number $L=S$) is a good quantum number and only ${}^1S_0$, ${}^1D_2$, ${}^1G_4$, $\cdots$ contribute. Note that the orbital angular-momentum $\ell = \lambda$ since $J=0$. Even in the β channel, we have the relationship $\ell_1 = \ell_2$ because of the parity conservation. (The spin value is $S=0$ and 1.) In the jj-coupling scheme, $j_1 = j_2 = \ell \pm 1/2$, which we denote as I. These restrictions are of course the result of the channel truncation that we don't include possible $\Xi N\alpha$ and $\Sigma\Sigma\alpha$ configurations in addition to the $\Lambda\Lambda\alpha$ one. The transformations from the LS-coupling scheme to the jj-coupling scheme are given by

$$\begin{aligned} \langle \widehat{\boldsymbol{p}}_3\widehat{\boldsymbol{q}}_3;123|\gamma\rangle_{jj} &= (-)^S\langle \widehat{\boldsymbol{p}}_3\widehat{\boldsymbol{q}}_3;123|\gamma\rangle_{LS} \\ \langle \widehat{\boldsymbol{p}}_2\widehat{\boldsymbol{q}}_2;312|\beta\rangle_{jj} &= \sum_S (-)^{\frac{1}{2}+I+\ell_1+S}\widehat{I}\widehat{S}\begin{Bmatrix} \ell_1 & \frac{1}{2} & I \\ \frac{1}{2} & \ell_1 & S \end{Bmatrix}\langle \widehat{\boldsymbol{p}}_2\widehat{\boldsymbol{q}}_2;312|\beta\rangle_{LS} \end{aligned} \tag{3·7}$$

The coordinate transformation in momentum space is

$$\begin{aligned} \begin{pmatrix} \boldsymbol{p}_1 \\ \boldsymbol{q}_1 \end{pmatrix} &= \begin{pmatrix} -\frac{4}{4+\zeta} & -\frac{2+\zeta}{4+\zeta} \\ 1 & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} \boldsymbol{p}_3 \\ \boldsymbol{q}_3 \end{pmatrix} \\ \begin{pmatrix} \boldsymbol{p}_2 \\ \boldsymbol{q}_2 \end{pmatrix} &= \begin{pmatrix} -\frac{\zeta}{4+\zeta} & -\frac{8(2+\zeta)}{(4+\zeta)^2} \\ 1 & -\frac{\zeta}{4+\zeta} \end{pmatrix}\begin{pmatrix} \boldsymbol{p}_1 \\ \boldsymbol{q}_1 \end{pmatrix} \\ &= \begin{pmatrix} -\frac{4}{4+\zeta} & \frac{2+\zeta}{4+\zeta} \\ -1 & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} \boldsymbol{p}_3 \\ \boldsymbol{q}_3 \end{pmatrix} \end{aligned} \tag{3·8}$$

The total wave function is given by

$$\begin{aligned} \Psi(\boldsymbol{p}_3,\boldsymbol{q}_3) &= u(\boldsymbol{p}_3,\boldsymbol{q}_3) - v(-\boldsymbol{p}_1,\boldsymbol{q}_1) + v(\boldsymbol{p}_2,\boldsymbol{q}_2) \\ &\qquad \text{with} \qquad u(-\boldsymbol{p}_3,\boldsymbol{q}_3) = -u(\boldsymbol{p}_3,\boldsymbol{q}_3) \end{aligned} \tag{3·9}$$

The three-body kinetic energy term is expressed as

$$H_0 = \frac{\hbar^2}{M_N}\frac{1}{\zeta}\left(\boldsymbol{p}_3{}^2 + \frac{2+\zeta}{8}\boldsymbol{q}_3{}^2\right) = \frac{\hbar^2}{M_N}\frac{1}{\zeta}\left(\frac{4+\zeta}{8}\boldsymbol{p}_2{}^2 + \frac{2+\zeta}{4+\zeta}\boldsymbol{q}_2{}^2\right) \tag{3·10}$$

The partial wave decomposition of the Faddeev equation is given by

$$\begin{aligned}
u_\gamma(p,q) &= \left[E - \frac{\hbar^2}{M_N}\left(p^2 + \frac{2+\zeta}{8}q^2\right)\right]^{-1}\int_0^\infty q'^2dq'\int_{-1}^1 dx \\
&\times\langle p|\widetilde{T}_{(\lambda S)\lambda}\left(E - \frac{\hbar^2}{M_N}\frac{2+\zeta}{8\zeta}q^2,\varepsilon_\gamma\right)|p_1\rangle\sum_\beta \frac{1}{{p_1}^\lambda}g_{\gamma\beta}(q,q';x)\frac{1}{{p_2}^{\ell_1}}v_\beta(p_2,q') \\
v_\beta(p,q) &= \left[E - \frac{\hbar^2}{M_N}\left(\frac{4+\zeta}{8}p^2 + \frac{2+\zeta}{4+\zeta}q^2\right)\right]^{-1}\frac{1}{2}\int_0^\infty q'^2dq'\int_{-1}^1 dx \\
&\times\left\{\langle p|T_{\ell_1}\left(E - \frac{\hbar^2}{M_N}\frac{2+\zeta}{\zeta(4+\zeta)}q^2,\varepsilon_\beta\right)|{p_1}'\rangle\sum_\gamma\frac{1}{{p_1'}^{\ell_1}}g_{\beta\gamma}(q,q';x)\frac{1}{{p_2'}^\lambda}u_\gamma(p_2',q')\right. \\
&\left.-\langle p|T_{\ell_1}\left(E - \frac{\hbar^2}{M_N}\frac{2+\zeta}{\zeta(4+\zeta)}q^2,\varepsilon_\beta\right)|\widetilde{p_1}\rangle\sum_{\beta'}\frac{1}{{\widetilde{p_1}}^{\ell_1}}g_{\beta\beta'}(q,q';x)\frac{1}{{\widetilde{p_2}}^{\ell_1'}}v_{\beta'}(\widetilde{p_2},q')\right\}
\end{aligned} \tag{3·11}$$

where

$$\left\{\begin{array}{c} p_1 = p\left(q',\frac{1}{2}q;x\right) \\ p_2 = p\left(q,\frac{4}{4+\zeta}q';x\right)\end{array}\right. \qquad \left\{\begin{array}{c} p_1' = p\left(q',\frac{4}{4+\zeta}q;x\right) \\ p_2' = p\left(q,\frac{1}{2}q';x\right)\end{array}\right. \qquad \left\{\begin{array}{c} \widetilde{p}_1 = p\left(q',\frac{\zeta}{4+\zeta}q;x\right) \\ \widetilde{p}_2 = p\left(q,\frac{\zeta}{4+\zeta}q';x\right)\end{array}\right. \tag{3·12}$$

with $p(q,q';x) = \sqrt{q^2+{q'}^2+2qq'x}$. In the jj-coupling scheme, the $\Lambda\alpha$ T-matrix in the second equation of Eq. (3·11) should be modified to $T_{\ell_1} \to T_{(\ell_1 1/2)I}$. The rearrangement factors are given by

$$\begin{aligned}
&g_{\gamma\beta}(q,q';x) = g_{\beta\gamma}(q',q;x) \\
&= \sum_{\lambda_1+\lambda_2=\lambda}\sum_{\lambda_1'+\lambda_2'=\ell_1} q^{\lambda_1'+\lambda_2}{q'}^{\lambda_1+\lambda_2'}\left(\frac{1}{2}\right)^{\lambda_2}\left(\frac{4}{4+\zeta}\right)^{\lambda_2'}\sum_k(2k+1)\ g_{\gamma\beta}^{\lambda_1\lambda_1'k}\ P_k(x) \\
&g_{\gamma\beta}^{\lambda_1\lambda_1'k} = (-)^\lambda G^{\lambda_1\lambda_1'kS}_{(\lambda\lambda),(\ell_1\ell_1)} \qquad\qquad \text{in } LS\text{−coupling} \\
&= (-1)^{\frac{1}{2}+I+\lambda+\ell_1}\widehat{I}\widehat{S}\left\{\begin{array}{ccc}\ell_1 & \frac{1}{2} & I \\ \frac{1}{2} & \ell_1 & S\end{array}\right\}G^{\lambda_1\lambda_1'kS}_{(\lambda\lambda),(\ell_1\ell_1)} \qquad \text{in } jj\text{−coupling}
\end{aligned} \tag{3·13}$$

and

$$\begin{aligned}
&g_{\beta\beta'}(q,q';x) \\
&= \sum_{\lambda_1+\lambda_2=\ell_1}\sum_{\lambda_1'+\lambda_2'=\ell_1'} q^{\lambda_1'+\lambda_2}{q'}^{\lambda_1+\lambda_2'}\left(\frac{\zeta}{4+\zeta}\right)^{\lambda_2+\lambda_2'}\sum_k(2k+1)\ g_{\beta\beta'}^{\lambda_1\lambda_1'k}\ P_k(x) \\
&g_{\beta\beta'}^{\lambda_1\lambda_1'k} = (-1)^{1-S+\ell_1+{\ell_1}'}\delta_{S,S'}G^{\lambda_1\lambda_1'kS}_{(\ell_1\ell_1),({\ell_1}'{\ell_1}')} \qquad\qquad \text{in } LS\text{−coupling} \\
&= \sum_S(-1)^{I+I'+S}\widehat{I}\widehat{I'}\left(\widehat{S}\right)^2\left\{\begin{array}{ccc}\ell_1 & \frac{1}{2} & I \\ \frac{1}{2} & \ell_1 & S\end{array}\right\}\left\{\begin{array}{ccc}{\ell_1}' & \frac{1}{2} & I' \\ \frac{1}{2} & {\ell_1}' & S\end{array}\right\}G^{\lambda_1\lambda_1'kS}_{(\ell_1\ell_1),({\ell_1}'{\ell_1}')} \\
&\qquad\qquad \text{in } jj\text{−coupling}
\end{aligned} \tag{3·14}$$

The expressions for ε_γ and ε_β in Eq. (2·26) and Eq. (2·27) are valid except for a trivial change of the kinetic energy terms of Eq. (3·10) and the phase change from boson to fermion systems in Eq. (2·25). Here we only give an explicit expression for $\langle\Psi|h_\gamma|\Psi\rangle$:

$$\begin{aligned}
\langle\Psi|h_\gamma|\Psi\rangle &= \sum_\gamma \int_0^\infty p^2 dp q^2 dq \frac{\hbar^2}{M_N}\frac{1}{\zeta} p^2 \left(u_\gamma(p,q)\right)^2 \\
&+ \sum_{\gamma,\beta}\int_0^\infty q^2 dq q'^2 dq' \int_{-1}^1 dx u_\gamma(p_1,q) 2\frac{\hbar^2}{M_N}\frac{1}{\zeta} {p_1}^2 \frac{1}{{p_1}^\lambda} g_{\gamma\beta}(q,q';x)\frac{1}{{p_2}^{\ell_1}} v_\beta(p_2,q') \\
&+ \sum_{\beta,\beta'}\int_0^\infty p^2 dp q^2 dq v_\beta(p,q)\frac{\hbar^2}{2M_N}\frac{1}{\zeta}\left\{\left[p^2+\left(\frac{2(2+\zeta)}{4+\zeta}\right)^2 q^2\right]\delta_{\beta,\beta'} + \frac{4(2+\zeta)}{4+\zeta} pq f_{\beta\beta'}\right\} \\
&\times v_{\beta'}(p,q) \\
&- \sum_{\beta,\beta'}\int_0^\infty q^2 dq q'^2 dq' \int_{-1}^1 dx v_\beta(\widetilde{p_1},q)\frac{\hbar^2}{4M_N}\frac{1}{\zeta}\left(q^2+q'^2-2qq'x\right)\frac{1}{{\widetilde{p_1}}^{\ell_1}} \\
&\times g_{\beta\beta'}(q,q';x)\frac{1}{{\widetilde{p_2}}^{\ell_1'}} v_{\beta'}(\widetilde{p_2},q') \qquad (3\cdot15)
\end{aligned}$$

Here, $f_{\beta\beta'}$ is given by

$$\begin{aligned}
f_{\beta\beta'} &= (-1)^{\ell_1+\ell_1'+S}\widehat{\ell_1}^2\langle\ell_1 0 1 0|\ell_1'\rangle^2 \begin{Bmatrix} \ell_1 & \ell_1 & S \\ \ell_1' & \ell_1' & 1 \end{Bmatrix} \qquad \text{in } LS\text{−coupling} \\
&= (-1)^{I-I'}\widehat{\ell_1}^2\widehat{I}\widehat{I'}\langle\ell_1 0 1 0|\ell_1'\rangle^2 \sum_S (-1)^S(\widehat{S})^2 \begin{Bmatrix} \ell_1 & \frac{1}{2} & I \\ \frac{1}{2} & \ell_1 & S \end{Bmatrix}\begin{Bmatrix} \ell_1' & \frac{1}{2} & I' \\ \frac{1}{2} & \ell_1' & S \end{Bmatrix} \\
&\quad\times \begin{Bmatrix} \ell_1 & \ell_1 & S \\ \ell_1' & \ell_1' & 1 \end{Bmatrix} \qquad \text{in } jj\text{−coupling} \qquad (3\cdot16)
\end{aligned}$$

Expressions for numerical calculations

(Faddeev equation)

$$\begin{aligned}
u_{\gamma i\mu} &= \sum_k (G\widetilde{T})_{\lambda ik\mu}\sum_{\nu\kappa} S^{(1)}_{k\nu\mu\kappa}\sum_\beta B_{\gamma\mu,\beta\nu,\kappa}\sum_j S^{(2)}_{j\mu\nu\kappa}\ v_{\beta j\nu} \\
v_{\beta i\mu} &= \sum_k (GT)_{\ell_1 ik\mu}\sum_{\nu\kappa}\left\{S^{(2)}_{k\nu\mu\kappa}\sum_\alpha \frac{1}{2}B_{\alpha\nu,\beta\mu,\kappa}\sum_j S^{(1)}_{j\mu\nu\kappa}\ u_{\alpha j\nu}\right. \\
&\qquad \left. - S^{(3)}_{k\nu\mu\kappa}\sum_{\beta'} B_{\beta\mu,\beta'\nu,\kappa}\sum_j S^{(3)}_{j\mu\nu\kappa}\ v_{\beta' j\nu}\right\} \qquad (3\cdot17)
\end{aligned}$$

where

$$u_{\gamma i\mu} = u_\gamma(p_i,q_\mu)q_\mu\sqrt{w_\mu} \qquad \text{and} \qquad v_{\beta i\mu} = v_\beta(p_i,q_\mu)q_\mu\sqrt{w_\mu} \qquad (3\cdot18)$$

and

$$(G\widetilde{T})_{\lambda ik\mu} = \left[E - \frac{\hbar^2}{M_N}\frac{1}{\zeta}\left({p_i}^2 + \frac{2+\zeta}{8}{q_\mu}^2\right)\right]^{-1} \frac{4\pi}{(2\pi)^3}\widetilde{T}_{(\lambda S)\lambda}\left(p_i, p_k; E - \frac{\hbar^2}{M_N}\frac{1}{\zeta}\frac{2+\zeta}{8}{q_\mu}^2, \varepsilon_\gamma\right)$$

$$(GT)_{\ell_1 ik\mu} = \left[E - \frac{\hbar^2}{M_N}\frac{1}{\zeta}\left(\frac{4+\zeta}{8}{p_i}^2 + \frac{2+\zeta}{4+\zeta}{q_\mu}^2\right)\right]^{-1}$$
$$\times\frac{4\pi}{(2\pi)^3}T_{\ell_1}\left(p_i, p_k; E - \frac{\hbar^2}{M_N}\frac{1}{\zeta}\frac{2+\zeta}{4+\zeta}{q_\mu}^2, \varepsilon_\beta\right) \tag{3·19}$$

and

$$S^{(n)}_{i\mu\nu\kappa} = S_i(p^{(n)}_{\mu\nu\kappa}) \qquad (n = 1 \sim 3) \tag{3·20}$$

with

$$\begin{aligned} p^{(1)}_{\mu\nu\kappa} &= p\left(q_\mu, \frac{1}{2}q_\nu, x_\kappa\right) \\ p^{(2)}_{\mu\nu\kappa} &= p\left(q_\mu, \frac{4}{4+\zeta}q_\nu, x_\kappa\right) \\ p^{(3)}_{\mu\nu\kappa} &= p\left(q_\mu, \frac{\zeta}{4+\zeta}q_\nu, x_\kappa\right) \end{aligned} \tag{3·21}$$

and $p(q, q', x) = \sqrt{q^2 + {q'}^2 + 2qq'x}$. The rearrangement coefficients are given by

$$B_{\gamma\mu,\beta\nu,\kappa} = \left(\frac{1}{p^{(1)}_{\nu\mu\kappa}}\right)^{\lambda} g_{\gamma\beta}(q_\mu, q_\nu; x_\kappa)\left(\frac{1}{p^{(2)}_{\mu\nu\kappa}}\right)^{\ell_1} q_\mu q_\nu\sqrt{w_\mu w_\nu}w_\kappa$$

$$B_{\beta\mu,\gamma\nu,\kappa} = \frac{1}{2}B_{\gamma\nu,\beta\mu,\kappa} \tag{3·22}$$

$$B_{\beta\mu,\beta'\nu,\kappa} = \frac{1}{2}\left(\frac{1}{p^{(3)}_{\nu\mu\kappa}}\right)^{\ell_1} g_{\beta\beta'}(q_\mu, q_\nu; x_\kappa)\left(\frac{1}{p^{(3)}_{\mu\nu\kappa}}\right)^{\ell_1{}'} q_\mu q_\nu\sqrt{w_\mu w_\nu}w_\kappa \tag{3·23}$$

where $g_{\beta\gamma}(q_\mu, q_\nu; x_\kappa) = g_{\gamma\beta}(q_\nu, q_\mu; x_\kappa)$.

(ε_γ and ε_β)

$$\langle\psi_\gamma|\begin{Bmatrix} 1 \\ H_0 \end{Bmatrix}|\Psi\rangle = \sum_{i\mu}\begin{Bmatrix} 1 \\ \frac{\hbar^2}{M_N}\frac{1}{\zeta}\left({p_i}^2 + \frac{2+\zeta}{8}{q_\mu}^2\right) \end{Bmatrix}\left(\sum_\gamma (u_{\gamma i\mu})^2\right){p_i}^2 w_i$$
$$+\sum_{\gamma\mu}\sum_{\nu\kappa}\left(\sum_i u_{\gamma i\mu}S^{(1)}_{i\nu\mu\kappa}\right)\begin{Bmatrix} 1 \\ \frac{\hbar^2}{M_N}\frac{1}{\zeta}\left({p^{(1)}_{\nu\mu\kappa}}^2 + \frac{2+\zeta}{8}{q_\mu}^2\right) \end{Bmatrix}\sum_\beta B_{\gamma\mu,\beta\nu,\kappa}\sum_j S^{(2)}_{j\mu\nu\kappa}v_{\beta j\nu}$$

$$\langle\psi_\beta|\begin{Bmatrix} 1 \\ H_0 \end{Bmatrix}|\Psi\rangle = \sum_{i\mu}\begin{Bmatrix} 1 \\ \frac{\hbar^2}{M_N}\frac{1}{\zeta}\left(\frac{4+\zeta}{8}{p_i}^2 + \frac{2+\zeta}{4+\zeta}{q_\mu}^2\right) \end{Bmatrix}\left(\sum_\beta (v_{\beta i\mu})^2\right){p_i}^2 w_i$$
$$-\sum_{\beta\mu}\sum_{\nu\kappa}\left(\sum_i v_{\beta i\mu}S^{(3)}_{i\nu\mu\kappa}\right)\begin{Bmatrix} 1 \\ \frac{\hbar^2}{M_N}\frac{1}{\zeta}\left(\frac{4+\zeta}{8}{p^{(3)}_{\nu\mu\kappa}}^2 + \frac{2+\zeta}{4+\zeta}{q_\mu}^2\right) \end{Bmatrix}\sum_{\beta'} B_{\beta\mu,\beta'\nu,\kappa}\sum_j S^{(3)}_{j\mu\nu\kappa}v_{\beta' j\nu}$$

$$
+\frac{1}{2}\sum_{\gamma\mu}\sum_{\nu\kappa}\left(\sum_i u_{\gamma i\mu}S^{(1)}_{i\nu\mu\kappa}\right)\left\{\ \frac{1}{\frac{\hbar^2}{4M_N}\left({p^{(1)}_{\nu\mu\kappa}}^2+\frac{8+\zeta}{4\zeta}{q_\mu}^2\right)}\ \right\}\sum_\beta B_{\gamma\mu,\beta\nu,\kappa}\sum_j S^{(2)}_{j\mu\nu\kappa}v_{\beta j\nu}\ \ ,
$$

$$
\langle\Psi|h_\gamma|\Psi\rangle=\frac{\hbar^2}{M_N}\frac{1}{\zeta}\sum_{i\mu}\left(\sum_\gamma(u_{\gamma i\mu})^2\right){p_i}^4w_i+\frac{\hbar^2}{M_N}\frac{1}{\zeta}\sum_{i\mu}\left\{\left[{p_i}^2+\left(\frac{2(2+\zeta)}{4+\zeta}\right)^2{q_\mu}^2\right]\right.
$$

$$
\left.\times\left(\sum_\beta(v_{\beta i\mu})^2\right)+\frac{4(2+\zeta)}{4+\zeta}p_iq_\mu\left(\sum_{\beta\beta'}v_{\beta i\mu}f_{\beta\beta'}v_{\beta' i\mu}\right)\right\}{p_i}^2w_i
$$

$$
+2\frac{\hbar^2}{M_N}\frac{1}{\zeta}\sum_{\gamma\mu}\sum_{\nu\kappa}\left(\sum_i u_{\gamma i\mu}S^{(1)}_{i\nu\mu\kappa}\right){p^{(1)}_{\nu\mu\kappa}}^2\sum_\beta B_{\gamma\mu,\beta\nu,\kappa}\sum_j S^{(2)}_{j\mu\nu\kappa}v_{\beta j\nu}
$$

$$
+\frac{\hbar^2}{2M_N}\frac{1}{\zeta}\sum_{\beta\mu}\sum_{\nu\kappa}\left(\sum_i v_{\beta i\mu}S^{(3)}_{i\nu\mu\kappa}\right)\left({q_\mu}^2+{q_\nu}^2-2q_\mu q_\nu x_\kappa\right)\sum_{\beta'}B_{\beta\mu,\beta'\nu,\kappa}\sum_j S^{(3)}_{j\mu\nu\kappa}v_{\beta' j\nu}\ \ . \tag{3·24}
$$

ε_γ and ε_β are obtained from

$$
\begin{aligned}
\varepsilon_\gamma &= E\langle\psi_\gamma|\Psi\rangle-\langle\psi_\gamma|H_0|\Psi\rangle+\langle\Psi|h_\gamma|\Psi\rangle\\
\varepsilon_\beta &= E\langle\psi_\beta|\Psi\rangle+\frac{2+\zeta}{4+\zeta}\langle\psi_\gamma|H_0|\Psi\rangle+\frac{\zeta}{4+\zeta}\left[\langle\psi_\beta|H_0|\Psi\rangle-\langle\Psi|h_\gamma|\Psi\rangle\right]
\end{aligned}\tag{3·25}
$$

§4. ${}^{3}_{\Lambda}$H system

In this case, we choose the minus sign for $\pm$ in Eq. (1·24) - Eq. (1·27). u, v are vectors

$$
u=\begin{pmatrix}u_\Lambda\\ u_\Sigma\end{pmatrix}\ ,\qquad v=\begin{pmatrix}v_\Lambda\\ v_\Sigma\end{pmatrix}\tag{4·1}
$$

and T-matrices and the Green functions are matrices

$$
\begin{aligned}
t=t_{12}&=\begin{pmatrix}t^\Lambda_{NN}&0\\0&t^\Sigma_{NN}\end{pmatrix}\ ,\qquad \tau=(123)^2t_{13}(123)=\begin{pmatrix}\tau_{\Lambda\Lambda}&\tau_{\Lambda\Sigma}\\ \tau_{\Sigma\Lambda}&\tau_{\Sigma\Sigma}\end{pmatrix}\\
G_0&=\begin{pmatrix}G^\Lambda_0&0\\0&G^\Sigma_0\end{pmatrix}
\end{aligned}\tag{4·2}
$$

The NN T-matrix has two separate components

$$
\begin{aligned}
t^\Lambda_{NN}&=v_{NN}+v_{NN}\ G^\Lambda_0\ t^\Lambda_{NN}\\
t^\Sigma_{NN}&=v_{NN}+v_{NN}\ G^\Sigma_0\ t^\Sigma_{NN}\ ,
\end{aligned}\tag{4·3}
$$

while ΛN-ΣN T-matrix is the coupled one:

$$
\begin{pmatrix}\tau_{\Lambda\Lambda}&\tau_{\Lambda\Sigma}\\ \tau_{\Sigma\Lambda}&\tau_{\Sigma\Sigma}\end{pmatrix}=\begin{pmatrix}v_{\Lambda\Lambda}&v_{\Lambda\Sigma}\\ v_{\Sigma\Lambda}&v_{\Sigma\Sigma}\end{pmatrix}\begin{pmatrix}G^\Lambda_0&0\\0&G^\Sigma_0\end{pmatrix}\begin{pmatrix}\tau_{\Lambda\Lambda}&\tau_{\Lambda\Sigma}\\ \tau_{\Sigma\Lambda}&\tau_{\Sigma\Sigma}\end{pmatrix}\tag{4·4}
$$

The mass factors for ${}^{3}_{\Lambda}$H system is easily obtained from those for ${}^{9}_{\Lambda}$Be system. Setting

$$\zeta = \zeta_\Lambda = \frac{M_\Lambda}{M_N} \qquad \text{or} \qquad \zeta_\Sigma = \frac{M_\Sigma}{M_N} \ , \tag{4·5}$$

we only need to move $\zeta/4 \to \zeta_\Lambda$ or ζ_Σ and $4M_N \to M_N$. Then, by measuring the total energy E from the $2N + \Lambda$ threshold, we find

$$\begin{aligned} G_0^\Lambda &= \left[E - \frac{\hbar^2}{M_N} \left({\boldsymbol{p}_3}^2 + \frac{\zeta_\Lambda + 2}{4\zeta_\Lambda} {\boldsymbol{q}_3}^2 \right) \right]^{-1} \\ &= \left[E - \frac{\hbar^2}{2M_N} \left(\frac{\zeta_\Lambda + 1}{\zeta_\Lambda} {\boldsymbol{p}_2}^2 + \frac{\zeta_\Lambda + 2}{\zeta_\Lambda + 1} {\boldsymbol{q}_2}^2 \right) \right]^{-1} \\ G_0^\Sigma &= \left[E - \Delta M_{\Sigma\Lambda} - \frac{\hbar^2}{M_N} \left({\boldsymbol{p}_3}^2 + \frac{\zeta_\Sigma + 2}{4\zeta_\Sigma} {\boldsymbol{q}_3}^2 \right) \right]^{-1} \\ &= \left[E - \Delta M_{\Sigma\Lambda} - \frac{\hbar^2}{2M_N} \left(\frac{\zeta_\Sigma + 1}{\zeta_\Sigma} {\boldsymbol{p}_2}^2 + \frac{\zeta_\Sigma + 2}{\zeta_\Sigma + 1} {\boldsymbol{q}_2}^2 \right) \right]^{-1} \ , \end{aligned} \tag{4·6}$$

where $\Delta M_{\Sigma\Lambda} = M_\Sigma - M_\Lambda$. For the NN T-matrix, Eq. (4·3) is expressed by the NN 2-body T-matrix as

$$t_{NN}^{\Lambda,\Sigma} = t_{NN}^{(2)} \left(E - \frac{\hbar^2}{M_N} \frac{\xi_{\Lambda,\Sigma} + 2}{4\xi_{\Lambda,\Sigma}} \boldsymbol{q}^2, \varepsilon \right) \tag{4·7}$$

However, ΛN-ΣN T-matrix in the 3-body space cannot be reduced to the 2-body T-matrix in the free space. In the ΛN channel, the starting energies are $E - \frac{\hbar^2}{2M_N} \frac{\xi_\Lambda + 2}{\xi_\Lambda + 1} \boldsymbol{q}^2$, while in the ΣN channel, it is $E - \Delta M_{\Sigma\Lambda} - \frac{\hbar^2}{2M_N} \frac{\xi_\Sigma + 2}{\xi_\Sigma + 1} \boldsymbol{q}^2$. These are not expressed as the form of E and $E - \Delta M_{\Sigma\Lambda}$ in the free space scattering, because of the mass difference of Λ and Σ. This is one source of the many-body effects to the bare 2-baryon interaction, but is not a unique source. There are many other effects which are not taken into account in the bare two-baryon interaction, since the usual formulation of the baryon-baryon interaction assumes the c.m. system to construct the interaction model. Since this difference is $\boldsymbol{q}^2$ dependent, it is not easy to take into account without affecting the simple procedure of constructing the redundancy-free T-matrix $\widetilde{\tau}^{(2)}$. It also causes a serious problem, how to determine the ε values for Λ and Σ, separately. From these reasons, we simply assume that the energy dependence of the quark-model kernel is determined solely from the ΛN channel (i.e., $\varepsilon_{\Sigma N} = \varepsilon_{\Lambda N} - \Delta E_{\Sigma\Lambda}$), and use

$$\tau_{YN} = \widetilde{\tau}^{(2)} \left(E - \Delta E_\beta - \frac{\hbar^2}{2M_N} \frac{\zeta_\beta + 2}{\zeta_\beta + 1} \boldsymbol{q}^2, \varepsilon_\beta \right) \ , \tag{4·8}$$

generated from $V_{\rm RGM}^{\rm mod}(\varepsilon)$ in Eq. (4·36).

Various channels of the ${}^{3}_{\Lambda}$H system are specified by

$$|p_3 q_3 \gamma\rangle = |p_3 q_3; [(\lambda s) I, (\ell \tfrac{1}{2}) j] \tfrac{1}{2}; [(\tfrac{1}{2}\tfrac{1}{2})\, t, t]\, 0\rangle \qquad \text{with} \qquad \gamma = (\lambda s) I \ell j t$$

$$
\begin{aligned}
&\langle \widehat{\boldsymbol{p}}_3\widehat{\boldsymbol{q}}_3;123|\gamma\rangle = \left[\mathcal{Y}_{(\lambda s)I}(\widehat{\boldsymbol{p}}_3;12)\mathcal{Y}_{(\ell\frac{1}{2})j}(\widehat{\boldsymbol{q}}_3;3)\right]_{\frac{1}{2}J_z}\left[\eta_{(\frac{1}{2}\frac{1}{2})t}(12)\eta_t(3)\right]_0\\
&\langle 123|S0;\gamma\rangle = \left[\chi_{(\frac{1}{2}\frac{1}{2})s}(12)\chi_{\frac{1}{2}}(3)\right]_S\left[\eta_{(\frac{1}{2}\frac{1}{2})t}(12)\eta_t(3)\right]_0\\
&|p_2q_2\beta\rangle = |p_2q_2;[(\ell_1s_1)I_1,(\ell_2\tfrac{1}{2})j_1]\,\tfrac{1}{2};[(t\tfrac{1}{2})\,\tfrac{1}{2},\tfrac{1}{2}]\,0\rangle \qquad \text{with} \qquad \beta=(\ell_1s_1)I_1\ell_2j_1t\\
&\langle \widehat{\boldsymbol{p}}_2\widehat{\boldsymbol{q}}_2;312|\beta\rangle = \left[\mathcal{Y}_{(\ell_1s_1)I_1}(\widehat{\boldsymbol{p}}_2;31)\mathcal{Y}_{(\ell_2\frac{1}{2})j_1}(\widehat{\boldsymbol{q}}_2;2)\right]_{\frac{1}{2}J_z}\left[\eta_{(t\frac{1}{2})\frac{1}{2}}(31)\eta_{\frac{1}{2}}(2)\right]_0\\
&\langle 312|S0;\beta\rangle = \left[\chi_{(\frac{1}{2}\frac{1}{2})s_1}(31)\chi_{\frac{1}{2}}(2)\right]_S\left[\eta_{(t\frac{1}{2})t}(31)\eta_{\frac{1}{2}}(2)\right]_0
\end{aligned}
\tag{4·9}
$$

In the γ channel, we have the restriction of the quantum numbers through the anti-symmetric property of two nucleons through $(-)^{\lambda+s+t}=-1$, in addition to the parity conservation $(-)^{\lambda+\ell}=(-)^{\ell_1+\ell_2}=1$. The choice of the Λ and Σ channels is uniquely specified by the isospin value of $t=0$ or 1.

Faddeev equation

The Faddeev equation of the ${}^3_\Lambda$H system are easily obtained from Eq. (2·11) - Eq. (2·12), by just a small change of the mass factors, quantum numbers and the T-matrices:

$$
\begin{aligned}
&u_\gamma(p,q) = \left[E-\Delta E_\gamma - \frac{\hbar^2}{M_N}\left(p^2+\frac{\zeta_\gamma+2}{4\zeta_\gamma}q^2\right)\right]^{-1}\int_0^\infty q'^2dq'\int_{-1}^1 dx\\
&\times{\sum_{\gamma'}}'\langle p|t^{sI;t}_{\lambda\lambda'}\left(E-\Delta E_\gamma-\frac{\hbar^2}{M_N}\frac{\zeta_\gamma+2}{4\zeta_\gamma}q^2,\varepsilon_\gamma\right)|p_1\rangle\sum_\beta\frac{1}{p_1{}^{\lambda'}}g_{\gamma'\beta}(q,q';x)\frac{1}{p_2{}^{\ell_1}}v_\beta(p_2,q')\\
&v_\beta(p,q) = \left[E-\Delta E_\beta-\frac{\hbar^2}{2M_N}\left(\frac{\zeta_\beta+1}{\zeta_\beta}p^2+\frac{\zeta_\beta+2}{\zeta_\beta+1}q^2\right)\right]^{-1}\frac{1}{2}\int_0^\infty q'^2dq'\int_{-1}^1 dx\\
&\times{\sum_{\beta'}}'\left\{\langle p|\widetilde{\tau}_{(\ell_1s_1)I_1t;(\ell_1's_1')I_1t'}\left(E-\Delta E_\beta-\frac{\hbar^2}{2M_N}\frac{\zeta_\beta+2}{\zeta_\beta+1}q^2,\varepsilon_\beta\right)|p_1'\rangle\right.\\
&\times\sum_\gamma\frac{1}{p_1'{}^{\ell_1'}}g_{\beta'\gamma}(q,q';x)\frac{1}{p_2'{}^{\lambda}}u_\gamma(p_2',q')\\
&-\langle p|\widetilde{\tau}_{(\ell_1s_1)I_1t;(\ell_1's_1')I_1t'}\left(E-\Delta E_\beta-\frac{\hbar^2}{2M_N}\frac{\zeta_\beta+2}{\zeta_\beta+1}q^2,\varepsilon_\beta\right)|\widetilde{p}_1'\rangle\\
&\left.\times\sum_{\beta''}\frac{1}{\widetilde{p}_1{}^{\ell_1'}}g_{\beta'\beta''}(q,q';x)\frac{1}{\widetilde{p}_2{}^{\ell_1''}}v_{\beta''}(\widetilde{p}_2,q')\right\}
\end{aligned}
\tag{4·10}
$$

where

$$
\left\{\begin{array}{c} p_1=p\left(q',\frac{1}{2}q;x\right)\\ p_2=p\left(q,\frac{\zeta}{\zeta+1}q';x\right)\end{array}\right. \qquad
\left\{\begin{array}{c} p_1'=p\left(q',\frac{\zeta}{\zeta+1}q;x\right)\\ p_2'=p\left(q,\frac{1}{2}q';x\right)\end{array}\right. \qquad
\left\{\begin{array}{c} \widetilde{p}_1=p\left(q',\frac{1}{\zeta+1}q;x\right)\\ \widetilde{p}_2=p\left(q,\frac{1}{\zeta+1}q';x\right)\end{array}\right. \tag{4·11}
$$

with $p(q,q';x)=\sqrt{q^2+q'^2+2qq'x}$.

The rearrangement factors are obtained by applying the formula in Eq. (1·44) to the spin-isospin functions in Eq. (4·9):

$$
g_{\beta\gamma}(q,q';x) = g_{\gamma\beta}(q',q;x)
$$

$$= \sum_{\lambda_1+\lambda_2=\ell_1} \sum_{\lambda_1'+\lambda_2'=\lambda} q^{\lambda_1'+\lambda_2} q'^{\lambda_1+\lambda_2'} \left(\frac{\zeta}{\zeta+1}\right)^{\lambda_2} \left(\frac{1}{2}\right)^{\lambda_2'} \sum_k (2k+1)\ g^{\lambda_1\lambda_1' k}_{\beta\gamma}\ P_k(x)\ , \quad (4\cdot12)$$

where

$$g^{\lambda_1\lambda_1' k}_{\beta\gamma} = \delta_{t,t'}\ g^{\lambda_1\lambda_1' k}_{(\ell_1 s_1) I_1 (\ell_2 \frac{1}{2}) j_1, (\lambda s) I (\ell \frac{1}{2}) j} \quad , \quad (4\cdot13)$$

with

$$g^{\lambda_1\lambda_1' k}_{(\ell_1 s_1) I_1 (\ell_2 \frac{1}{2}) j_1, (\lambda s) I (\ell \frac{1}{2}) j}$$
$$= (-)^{\lambda+s} \widehat{s_1}\widehat{s} \sum_{LS} \begin{bmatrix} \ell_1 & s_1 & I_1 \\ \ell_2 & \frac{1}{2} & j_1 \\ L & S & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \lambda & s & I \\ \ell & \frac{1}{2} & j \\ L & S & \frac{1}{2} \end{bmatrix} \begin{Bmatrix} \frac{1}{2} & S & s_1 \\ \frac{1}{2} & \frac{1}{2} & s \end{Bmatrix} G^{\lambda_1\lambda_1' kL}_{(\ell_1\ell_2),(\lambda\ell)} \quad (4\cdot14)$$

and

$$g_{\beta\beta'}(q, q'; x)$$
$$= \sum_{\lambda_1+\lambda_2=\ell_1} \sum_{\lambda_1'+\lambda_2'=\ell_1'} q^{\lambda_1'+\lambda_2} q'^{\lambda_1+\lambda_2'} \left(\frac{1}{\zeta+1}\right)^{\lambda_2+\lambda_2'} \sum_k (2k+1)\ g^{\lambda_1\lambda_1' k}_{\beta\beta'}\ P_k(x) \quad , \quad (4\cdot15)$$

where

$$g^{\lambda_1\lambda_1' k}_{\beta\beta'} = \delta_{t,t'}\ (-)^{\ell_1+s_1+t_1}\ g^{\lambda_1\lambda_1' k}_{(\ell_1 s_1) I_1 (\ell_2 \frac{1}{2}) j_1, (\ell_1' s_1') I_1' (\ell_2' \frac{1}{2}) j_1'} \quad (4\cdot16)$$

Note that these rearrangement factors are all diagonal for the Λ - Σ transition (so that, ζ is uniquely determined as ζ_Λ or ζ_Σ, depending on the t value).

Calculation of ε_γ and ε_β values

For the NN T-matrix, the basic NN RGM equation is

$$\left[\, \varepsilon_\gamma - h_\gamma - V^{\rm RGM}_\gamma(\varepsilon_\gamma) \,\right]\ \chi = 0 \quad . \quad (4\cdot17)$$

Since the denominator of the 3-body Green function is $E - H_0 = E - h_{\bar\gamma} - h_\gamma$, the 2-body T-matrix in the 3-body model space is obtained by using $E - h_{\bar\gamma}$ as the starting energy of the 2-body T-matrix equation. Here E is the 3-body energy. The equation to determine ε_γ is

$$\varepsilon_\gamma = \langle P\Psi |\, h_\gamma + V^{\rm RGM}_\gamma(\varepsilon_\gamma)\, | P\Psi \rangle \quad (4\cdot18)$$

or more explicitly

$$\varepsilon_{NN} = \langle \Psi_{\Lambda NN} |\, h_{NN} + V^{\rm RGM}_{NN}(\varepsilon_{NN})\, | \Psi_{\Lambda NN} \rangle + \{\, \Lambda \to \Sigma \,\} \quad . \quad (4\cdot19)$$

The 2-body T-matrix in the 3-body model space is expressed as

$$(t_{NN})_\gamma = t^{(2)}_{NN}\left(E - h_{\bar{\gamma}}, \varepsilon_\gamma\right) \tag{4·20}$$

or more explicitly

$$t^{\Lambda,\Sigma}_{NN} = t^{(2)}_{NN}\left(E - \frac{\hbar^2}{M_N}\frac{\zeta_{\Lambda,\Sigma}+2}{4\zeta_{\Lambda,\Sigma}}\boldsymbol{q}^2, \varepsilon_{NN}\right) \quad . \tag{4·21}$$

For the YN T-matrix, the situation is more complex, since

1) ΛN-ΣN coupling involves a complete Pauli forbidden state
2) The starting energy is different between ΛN and ΣN channels, in particular, due to the difference of the reduced masses;

$$E - \frac{\hbar^2}{2M_N}\frac{\zeta_\Lambda+2}{\zeta_\Lambda+1}\boldsymbol{q}^2 \neq E - \frac{\hbar^2}{2M_N}\frac{\zeta_\Sigma+2}{\zeta_\Sigma+1}\boldsymbol{q}^2 \quad . \tag{4·22}$$

We start from the ΛN-ΣN coupled-channel RGM equation which takes into account the internal-energy and reduced-mass corrections of the kinetic energy term (see Section 4 of the G-matrix paper, Prog. Theor. Phys. 104 (2000) 1025)

$$\Lambda\left[\,\varepsilon^{\rm exp} - h_0^{\rm exp} - V^{\rm RGM}(\varepsilon)\,\right]\,\Lambda\chi = 0 \tag{4·23}$$

or

$$\varepsilon^{\rm exp}\chi = \left[\,h_0^{\rm exp} + V^{\rm RGM}(\varepsilon) + \Delta G\,\right]\,\chi \quad , \tag{4·24}$$

where

$$\varepsilon = E - E_{\rm int} \quad , \qquad \varepsilon^{\rm exp} = E - E^{\rm exp}_{\rm int} \quad . \tag{4·25}$$

In Eq. (4·24), ΔG is determined from

$$\Lambda\left[\,\varepsilon^{\rm exp} - h_0^{\rm exp} - V^{\rm RGM}(\varepsilon)\,\right]\,\Lambda = \varepsilon^{\rm exp} - h_0^{\rm exp} - V^{\rm RGM}(\varepsilon) - \Delta G \tag{4·26}$$

as

$$\Delta G = \Lambda\left(\Delta E_{\rm int} + \Delta h_0\right)\Lambda - \left(\Delta E_{\rm int} + \Delta h_0\right) \tag{4·27}$$

with

$$\Delta h_0 = h_0^{\rm exp} - h_0 \quad , \qquad \Delta E_{\rm int} = \varepsilon - \varepsilon^{\rm exp} = E^{\rm exp}_{\rm int} - E_{\rm int} \quad . \tag{4·28}$$

In order to find the redundancy-free solution, we modify Eq. (4·23) as

$$E\,\chi = \Lambda\left[\,E^{\rm exp}_{\rm int} + h_0^{\rm exp} + V^{\rm RGM}(\varepsilon)\,\right]\,\Lambda\chi \quad , \tag{4·29}$$

the solution of which satisfies $\langle u|\chi\rangle = 0$ for $E \neq 0$. Here we generalize E to the 3-body energy, but setting

$$E\,\chi = \Lambda\left[\,E^{\rm exp}_{\rm int} + h_0^{\rm exp} + V^{\rm RGM}(\varepsilon_0)\,\right]\,\Lambda\chi + \varepsilon_{\Lambda N}|u\rangle\langle u|\chi\rangle \tag{4·30}$$

with $\varepsilon_0 = \varepsilon_{\Lambda N} - E_{\rm int}$ (matrix) is not good enough, since the kinetic energy of the spectator has the momentum dependence. We include this momentum-dependent term to the internal-energy contribution. The correct equation is

$$
\begin{aligned}
E\,\chi = \Lambda \left[E_{\rm int}^{\rm exp} + \frac{\hbar^2}{2M_N}\frac{\zeta_\beta+2}{\zeta_\beta+1}\boldsymbol{q}^2 + h_0^{\rm exp} + V^{\rm RGM}(\varepsilon_0) \right] \Lambda\chi \\
+ \left(\varepsilon_{\Lambda N} + \frac{\hbar^2}{2M_N}\frac{\zeta_\Lambda+2}{\zeta_\Lambda+1}\boldsymbol{q}^2 \right) |u\rangle\langle u|\chi\rangle
\end{aligned}
\tag{4·31}
$$

The solution of Eq. (4·31) satisfies

$$
\langle u|\chi\rangle = 0 \qquad \text{if} \qquad E \neq \varepsilon_{\Lambda N} + \frac{\hbar^2}{2M_N}\frac{\zeta_\Lambda+2}{\zeta_\Lambda+1}\boldsymbol{q}^2 \quad . \tag{4·32}
$$

We can rewrite Eq. (4·31) in more familiar form by using Eq. (4·26) with E being replaced by $\varepsilon_{\Lambda N}$:

$$
\Lambda \left[\varepsilon_{\Lambda N} - E_{\rm int}^{\rm exp} - h_0^{\rm exp} - V^{\rm RGM}(\varepsilon_0) \right] \Lambda = \varepsilon_{\Lambda N} - E_{\rm int}^{\rm exp} - h_0^{\rm exp} - V^{\rm RGM}(\varepsilon_0) - \Delta G \quad . \tag{4·33}
$$

The final result is

$$
E\,\chi = \left[E_{\rm int}^{\rm exp} + \frac{\hbar^2}{2M_N}\frac{\zeta_\beta+2}{\zeta_\beta+1}\boldsymbol{q}^2 + h_0^{\rm exp} + V^{\rm RGM}(\varepsilon_0) + \Delta G + \Delta G' \right] \chi \quad , \tag{4·34}
$$

where

$$
\begin{aligned}
&\Delta G' = \Lambda\Delta\varepsilon\Lambda - \Delta\varepsilon \qquad \text{with} \\
&\Delta\varepsilon = \frac{\hbar^2}{2M_N}\left(\frac{\zeta_\beta+2}{\zeta_\beta+1} - \frac{\zeta_\Lambda+2}{\zeta_\Lambda+1} \right)\boldsymbol{q}^2 = \frac{\hbar^2}{2M_N}\left(\frac{1}{\zeta_\beta+1} - \frac{1}{\zeta_\Lambda+1} \right)\boldsymbol{q}^2 \quad .
\end{aligned}
\tag{4·35}
$$

We write this equation as

$$
\begin{aligned}
&\omega\,\chi = \left[h_0^{\rm exp} + V_{\rm RGM}^{\rm mod}(\varepsilon) \right] \chi \qquad \text{with} \qquad V_{\rm RGM}^{\rm mod}(\varepsilon) = V^{\rm RGM}(\varepsilon_0) + \Delta G + \Delta G' \\
&\omega = E - \frac{\hbar^2}{2M_N}\frac{\zeta_\beta+2}{\zeta_\beta+1}\boldsymbol{q}^2 - E_{\rm int}^{\rm exp} \\
&\varepsilon = \varepsilon_{\Lambda N} - \Delta\varepsilon - E_{\rm int}^{\rm exp} = \varepsilon_{\Lambda N} - \frac{\hbar^2}{2M_N}\left(\frac{\zeta_\beta+2}{\zeta_\beta+1} - \frac{\zeta_\Lambda+2}{\zeta_\Lambda+1} \right)\boldsymbol{q}^2 - E_{\rm int}^{\rm exp} \quad .
\end{aligned}
\tag{4·36}
$$

The meaning of ε is very clear if we realize

$$
V_{\rm RGM}^{\rm mod}(\varepsilon) = (\varepsilon - h_0^{\rm exp}) - \Lambda\,(\varepsilon - h_0^{\rm exp})\,\Lambda + v(\varepsilon_0) \tag{4·37}
$$

with $v(\varepsilon_0) = \Lambda V^{\rm RGM}(\varepsilon_0)\Lambda = \Lambda V_{\rm RGM}^{\rm mod}(\varepsilon)\Lambda$. This can be easily proved by using

$$
\Lambda \left[\varepsilon_0 - h_0 - V^{\rm RGM}(\varepsilon_0) \right] \Lambda = \varepsilon_0 - h_0 - V^{\rm RGM}(\varepsilon_0) \quad . \tag{4·38}
$$

From the structure of Eq. (4·37), we can show

$$
\omega - h_0^{\rm exp} - V_{\rm RGM}^{\rm mod}(\varepsilon) = \Lambda\,[\,\omega - h_0^{\rm exp} - v(\varepsilon_0)\,]\,\Lambda + |u\rangle\,(\omega - \varepsilon)\,\langle u| \quad , \tag{4·39}
$$

since

$$\omega - \varepsilon = E - \varepsilon_{\Lambda N} - \frac{\hbar^2}{2M_N}\frac{\zeta_\Lambda + 2}{\zeta_\Lambda + 1}\boldsymbol{q}^2 < 0 \tag{4·40}$$

is channel-independent. It is also clear that the original equation contains a trivial solution $|u\rangle$ when $\omega = \varepsilon$. From Eq. (4·40), all the discussion on the RGM T-matrix, $\widetilde{T}(\omega, \varepsilon)$, in the T-RGM paper (Prog. Theor. Phys. 107 (2002) 745) follows. For the determination of $\varepsilon_{\Lambda N}$ or ε_0, we approximate

$$\begin{aligned}
\omega &= \begin{pmatrix} E - \frac{\hbar^2}{2M_N}\frac{\zeta_\Lambda+2}{\zeta_\Lambda+1}\boldsymbol{q}^2 & 0 \\ 0 & E - \frac{\hbar^2}{2M_N}\frac{\zeta_\Sigma+2}{\zeta_\Sigma+1}\boldsymbol{q}^2 - \Delta E_{\Sigma\Lambda} \end{pmatrix} \\
&\to \varepsilon_\beta = \begin{pmatrix} \varepsilon_{\Lambda N} & 0 \\ 0 & \varepsilon_{\Sigma N} \end{pmatrix} \qquad \text{with} \qquad \varepsilon_{\Sigma N} = \varepsilon_{\Lambda N} - \Delta E_{\Sigma\Lambda}
\end{aligned} \tag{4·41}$$

and calculate

$$\varepsilon_{YN} = \langle P\Psi |\, h_0 + V^{\rm mod}_{\rm RGM}(\varepsilon)\, | P\Psi \rangle \quad , \tag{4·42}$$

or more explicitly

$$\varepsilon_{YN} = \sum_{\beta,\beta'} \langle \Psi_\beta |\, h_\beta \delta_{\beta,\beta'} + \left(V^{\rm mod}_{\rm RGM}(\varepsilon)\right)_{\beta,\beta'} |\Psi_{\beta'}\rangle \quad . \tag{4·43}$$

Then, from

$$\begin{aligned}
&\varepsilon_{YN} = \langle P\Psi | \begin{pmatrix} \varepsilon_{\Lambda N} & 0 \\ 0 & \varepsilon_{\Sigma N} \end{pmatrix} | P\Psi \rangle = \varepsilon_{\Lambda N} \langle \Psi | \Psi \rangle^\Lambda + \varepsilon_{\Sigma N} \langle \Psi | \Psi \rangle^\Sigma \\
&\Delta E_{\Sigma\Lambda} = \varepsilon_{\Lambda N} - \varepsilon_{\Sigma N} \quad ,
\end{aligned} \tag{4·44}$$

we obtain

$$\begin{aligned}
\varepsilon_{\Lambda N} &= \varepsilon_{YN} + \Delta E_{\Sigma\Lambda} \langle \Psi | \Psi \rangle^\Sigma \\
\varepsilon_{\Sigma N} &= \varepsilon_{YN} - \Delta E_{\Sigma\Lambda} \langle \Psi | \Psi \rangle^\Lambda \quad .
\end{aligned} \tag{4·45}$$

We calculate ε_γ and ε_β by using Eq. (2·22) - Eq. (2·23), but this time, h_α etc. and H_0 are 2×2 diagonal matrices. We should note that h_α, h_β and h_γ are composed only of the kinetic-energy terms, but H_0 in the present case contains the mass term

$$\Delta E = \begin{pmatrix} 0 & 0 \\ 0 & \Delta E_{\Sigma\Lambda} \end{pmatrix} \quad , \tag{4·46}$$

where $\Delta E_{\Sigma\Lambda} = M_\Sigma - M_\Lambda$. We use the notation H_0' only for the kinetic-energy term. Almost all the equations in Eq. (2·22) - Eq. (2·27) are valid by changing H_0 to H_0'. We only need to change $\langle \Psi | V_\gamma | \Psi \rangle = \langle \psi_\gamma | E - H_0 | \Psi \rangle$ etc. to $\langle \Psi | V_\gamma | \Psi \rangle = \langle \psi_\gamma | E - H_0' - \Delta E | \Psi \rangle$ etc. This is equivalent to changing $E \to E - \Delta E$. Noting that the kinetic-energy matrix

elements contain only the diagonal part of the $NN\Lambda$-$NN\Sigma$ coupling, we can easily show (we choose $(\alpha\beta\gamma) = (123)$ and 3 is the Λ or Σ)

$$\begin{aligned}
\varepsilon_{NN} &= \varepsilon_\gamma = \{E\langle\psi_\gamma|\Psi\rangle - \langle\psi_\gamma|H_0'|\Psi\rangle + \langle\Psi|h_\gamma|\Psi\rangle\}^\Lambda + \{\cdots\}^\Sigma - \Delta E_{\Sigma\Lambda}\langle\psi_\gamma|\Psi\rangle^\Sigma \\
\varepsilon_{\Lambda N} &= \left\{E\langle\psi_\beta|\Psi\rangle + \frac{\zeta+2}{2(\zeta+1)}\langle\psi_\gamma|H_0'|\Psi\rangle + \frac{1}{\zeta+1}\left[\langle\psi_\beta|H_0'|\Psi\rangle - \langle\Psi|h_\gamma|\Psi\rangle\right]\right\}^\Lambda \\
&\quad + \{\cdots\}^\Sigma + \Delta E_{\Sigma\Lambda}\left[\langle\Psi|\Psi\rangle^\Sigma - \langle\psi_\beta|\Psi\rangle^\Sigma\right] \\
\varepsilon_{\Sigma N} &= \varepsilon_{\Lambda N} - \Delta E_{\Sigma\Lambda}
\end{aligned} \tag{4·47}$$

Each matrix elements is calculated for the $NN\Lambda$ contribution and $NN\Sigma$ contribution,separately. These are given as

$$\begin{aligned}
\langle\psi_\gamma|H_0'|\Psi\rangle &= \langle u|H_0'|u\rangle + 2\ \langle u|H_0'|(123)v\rangle \\
\langle\psi_\beta|H_0'|\Psi\rangle &= \langle u|H_0'|(123)v\rangle + \langle v|H_0'|v-(13)v\rangle \\
\langle\Psi|h_\gamma|\Psi\rangle &= \langle u|h|u\rangle + 4\ \langle u|h|(123)v\rangle + 2\ \langle v|(123)^2h(123)^{-2}|v-(13)v\rangle
\end{aligned} \tag{4·48}$$

with

$$\langle u|\begin{Bmatrix}1\\H_0'\\h\end{Bmatrix}|u\rangle = \sum_\gamma \int_0^\infty p^2dpq^2dq \begin{Bmatrix}1\\ \frac{\hbar^2}{M_N}\left(p^2+\frac{\zeta+2}{4\zeta}q^2\right)\\ \frac{\hbar^2}{M_N}p^2\end{Bmatrix}(u_\gamma(p,q))^2$$

$$\langle u|\begin{Bmatrix}1\\H_0'\\h\end{Bmatrix}|(123)v\rangle = \frac{1}{2}\sum_{\gamma,\beta}\int_0^\infty q^2dqq'^2dq'\int_{-1}^1 dx u_\gamma(p_1,q)\begin{Bmatrix}1\\ \frac{\hbar^2}{M_N}\left({p_1}^2+\frac{\zeta+2}{4\zeta}q^2\right)\\ \frac{\hbar^2}{M_N}{p_1}^2\end{Bmatrix}$$
$$\times\frac{1}{{p_1}^\lambda}g_{\gamma\beta}(q,q';x)\frac{1}{{p_2}^{\ell_1}}v_\beta(p_2,q')$$

$$\langle v|\begin{Bmatrix}1\\H_0'\end{Bmatrix}|v\rangle = \sum_\beta\int_0^\infty p^2dpq^2dq\begin{Bmatrix}1\\ \frac{\hbar^2}{2M_N}\left(\frac{\zeta+1}{\zeta}p^2+\frac{\zeta+2}{\zeta+1}q^2\right)\end{Bmatrix}(v_\beta(p,q))^2$$

$$\langle v|\begin{Bmatrix}1\\H_0'\end{Bmatrix}|(13)v\rangle = \frac{1}{2}\sum_{\beta,\beta'}\int_0^\infty q^2dqq'^2dq'\int_{-1}^1 dx v_\beta(\widetilde{p_1},q)\begin{Bmatrix}1\\ \frac{\hbar^2}{2M_N}\left(\frac{\zeta+1}{\zeta}\widetilde{p_1}^2+\frac{\zeta+2}{\zeta+1}q^2\right)\end{Bmatrix}$$
$$\times\frac{1}{\widetilde{p_1}^{\ell_1}}g_{\beta\beta'}(q,q';x)\frac{1}{\widetilde{p_2}^{\ell_1'}}v_{\beta'}(\widetilde{p_2},q')$$

$$\langle v|(123)^2h(123)^{-2}|v\rangle = \sum_{\beta,\beta'}\int_0^\infty p^2dpq^2dqv_\beta(p,q)\frac{\hbar^2}{4M_N}\left\{\left[p^2+\left(\frac{\zeta+2}{\zeta+1}\right)^2q^2\right]\delta_{\beta,\beta'}\right.$$
$$\left.+\frac{2(\zeta+2)}{\zeta+1}pqf_{\beta\beta'}\right\}\ v_{\beta'}(p,q)$$

$$\langle v|(123)^2h(123)^{-2}|(13)v\rangle = \frac{1}{2}\sum_{\beta,\beta'}\int_0^\infty q^2dqq'^2dq'\int_{-1}^1 dx v_\beta(\widetilde{p_1},q)\frac{\hbar^2}{4M_N}$$
$$\times\left(q^2+q'^2-2qq'x\right)\frac{1}{\widetilde{p_1}^{\ell_1}}g_{\beta\beta'}(q,q';x)\frac{1}{\widetilde{p_2}^{\ell_1'}}v_{\beta'}(\widetilde{p_2},q') \tag{4·49}$$

Here, $f_{\beta\beta'}$ is non-zero only when $t = t'$ and is given by

$$f_{\beta\beta'} = \delta_{s_1,s_1'} \sum_{LS} \left[\begin{array}{ccc} \ell_1 & s_1 & I_1 \\ \ell_2 & \frac{1}{2} & j_1 \\ L & S & \frac{1}{2} \end{array} \right] \left[\begin{array}{ccc} \ell_1' & s_1' & I_1' \\ \ell_2' & \frac{1}{2} & j_1' \\ L & S & \frac{1}{2} \end{array} \right] f^L_{(\ell_1\ell_2),(\ell_1'\ell_2')} \tag{4·50}$$

with

$$\begin{aligned} f^L_{(\ell_1\ell_2),(\ell_1'\ell_2')} &= \int d\widehat{\boldsymbol{p}}_2 d\widehat{\boldsymbol{q}}_2 \; Y^*_{(\ell_1\ell_2)L}(\widehat{\boldsymbol{p}}_2, \widehat{\boldsymbol{q}}_2) \; (\widehat{\boldsymbol{p}}_2 \cdot \widehat{\boldsymbol{q}}_2) \; Y_{(\ell_1'\ell_2')L}(\widehat{\boldsymbol{p}}_2, \widehat{\boldsymbol{q}}_2) \\ &= (-1)^{\ell_1+\ell_2'+L} \widehat{\ell_1} \widehat{\ell_2} \langle \ell_1 0 1 0 | \ell_1' \rangle \langle \ell_2 0 1 0 | \ell_2' \rangle \left\{ \begin{array}{ccc} \ell_1 & \ell_2 & L \\ \ell_2' & \ell_1' & 1 \end{array} \right\} \end{aligned} \tag{4·51}$$

Eq. (4·51) is the LS-coupling expression in Eq. (2·28).