A REALISTIC BARYON-BARYON INTERACTION IN THE SU_6 QUARK MODEL AND ITS APPLICATIONS TO FEW-BODY SYSTEMS

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The recent quark-model baryon-baryon interaction by the Kyoto-Niigata group is applied to the triton, hypertriton, $2\alpha\Lambda$ and $2\Lambda\alpha$ systems, in which a new three-cluster Faddeev formalism, using the 2-cluster resonating-group method (RGM) kernel, is developed for the exact treatment of the Pauli forbidden states between clusters.

1. THREE-CLUSTER FADDEEV FORMALISM USING THE TWO-CLUSTER RGM KERNEL

The QCD-inspired spin-flavor SU_6 quark model for the baryon-baryon interaction, proposed by the Kyoto-Niigata group, is a unified model for the complete baryon octet ($B_8 = N$, Λ, Σ and Ξ), which has achieved very accurate descriptions of the NN and YN interactions.[1,2,3] In particular, the nucleon-nucleon (NN) interaction of the most recent model fss2 [2] is accurate enough to compare with the modern realistic meson-exchange models. These quark-model interactions can be used for realistic calculations of few-baryon and few-cluster systems, once an appropriate three-body equation is developed for the pairwise interactions described by the RGM kernel. The desired 3-cluster equation should be able to deal with non-locality and energy-dependence intrinsically involved in the quark-exchange RGM kernel. Furthermore, quark-model descriptions of the hyperon nucleon (YN) and hyperon hyperon (YY) interactions in the full coupled-channel formalism sometimes involve a Pauli forbidden state, which excludes the most compact spatial configuration, resulting in strongly repulsive nature of the interactions in some particular channels. We have recently formulated a new 3-cluster equation which uses two-cluster RGM kernels explicitly.[4] This equation exactly eliminates 3-cluster redundant components by the orthogonality of the total wave function to the pairwise two-cluster Pauli-forbidden states. The explicit energy dependence inherent in the exchange RGM kernel is self-consistently determined. This equation is entirely equivalent to the Faddeev equation which uses a modified singularity-free T-matrix (which we call the RGM T-matrix) constructed from the two-cluster RGM kernel. We first applied this formalism to a 3-dineutron system and the 3α system, and obtained complete agreement between the Faddeev calculations and variational calculations which use the translationally invariant harmonic-oscillator (h.o.) basis.[4,5] For the 3α system, the input is the 3-range Minnesota force (MN force) with the exchange mixture $u = 0.946869$, together with the h.o. width parameter, $\nu = 0.257$ fm^{-2}, for the $(0s)^4$ α-clusters. The 2α phase shifts are nicely reproduced in the 2α RGM. We find that the 3α ground-state energies obtained by solving the present 3α Faddeev equations are only about $1.0 \sim 1.7$ MeV higher than those of the full microscopic 3α RGM calculations.[5]

2. TRITON AND HYPERTRITON FADDEEV CALCULATIONS

The present 3-cluster RGM formalism was applied to the Faddeev calculations of the $3N$ bound state [6] and the (ΛNN)-(ΣNN) system for the hypertriton, employing the off-shell T-matrices which are derived from the non-local and energy-dependent RGM kernel for our quark-model NN and YN interactions of fss2 and FSS. The model fss2 yields the triton energy $E(^3\mathrm{H}) = -8.519$ MeV in the 50 channel calculation, when the np interaction is employed for any NN pairs in the isospin basis.[7] The charge rms radii for ^{3}H and ^{3}He are also correctly reproduced. This triton binding energy is the closest to the experimental value, $B_t^{\mathrm{exp}} = 8.48$ MeV, among many Faddeev calculations employing modern realistic NN interactions. A characteristic description of the short range correlations in the quark model is essential to reproduce the large binding energy and the correct size of the three-nucleon bound state without reducing the D-state probability of the deuteron. For the hypertriton calculation, exact treatment of the ΛN-ΣN coupling and the resulting Pauli forbidden state with the SU_3 $(11)_s$ symmetry is very important to obtain precise results. In the final calculation with 150 ΛNN and ΣNN channels included, we find $B_\Lambda({}^3_\Lambda\mathrm{H}) = 289$ keV and the ΣNN component $P_{\Sigma NN} = 0.805\,\%$ for the fss2 prediction. For our previous model FSS, we obtain $B_\Lambda({}^3_\Lambda\mathrm{H}) = 878$ keV and $P_{\Sigma NN} = 1.36\,\%$. Since ${B_\Lambda}^{\mathrm{exp}} = 130 \pm 50$ keV, the fss2 result is slightly overbound, which implies that the ${}^1S_0$ attraction of the ΛN interaction is slightly too attractive in comparison to the ${}^3S_1$ attraction. From these results, we can extrapolate the desired difference of the ${}^1S_0$ and ${}^3S_1$ phase shifts at the maximum values. It turns out to be $3^\circ \sim 7^\circ$ more attractive in the ${}^1S_0$ state, which is consistent with the result in ref. [8] which uses simulated interactions of the Nijmegen models.

3. $2\alpha\Lambda$ FADDEEV CALCULATION FOR ${}^9_\Lambda$Be

The formalism is now applied to the $2\alpha\Lambda$ Faddeev calculation for ${}^9_\Lambda$Be, by using the 2α RGM kernel and $\Lambda\alpha$ folding potentials for various ΛN effective forces. The effective ΛN force, denoted by SB (Sparenberg-Baye potential) in Table 1, is constructed from the ${}^1S_0$ and ${}^3S_1$ phase shifts predicted by the YN sector of the model fss2 [3], by using an inversion method based on supersymmetric quantum mechanics.[9] These are simple 2-range Gaussian potentials which reproduce the low-energy behaviour of the ΛN phase shifts obtained by the full coupled-channel calculations. Since any central and single-channel effective ΛN forces lead to the well-known overbinding problem of ${}^5_\Lambda$He [10] by about 2 MeV (in the present case, it

Table 1. The ground-state energy $E_{\mathrm{gr}}(0^+)$ and the 2^+ excitation energy $E_{\mathrm{x}}(2^+)$ in MeV, calculated by solving the Faddeev equation for the $2\alpha\Lambda$ system. In the last column, ΔE is a simple estimate of the ℓs splitting for the $5/2^+$ and $3/2^+$ excited states, using the $\boldsymbol{P}$=0 Wigner transform of the fss2 and FSS ΛN LS interactions. The model fss2 involves an extra σ-meson contribution, which is indicated by "$+\sigma$".

$V_{\Lambda N}$	$E_{\mathrm{gr}}(0^+)$ (MeV)			$E_{\mathrm{x}}(2^+)$ (MeV)	ΔE (keV)	
	ours	Hiyama [11]	diff.		fss2	FSS
SB	-6.837	–	–	2.915	$103+\sigma$	164
NS	-6.742	-6.81	0.07	2.916	$97+\sigma$	154
ND	-7.483	-7.57	0.09	2.935	$106+\sigma$	169
NF	-6.906	-7.00	0.09	2.930	$86+\sigma$	138
JA	-6.677	-6.76	0.08	2.919	$86+\sigma$	138
JB	-6.474	-6.55	0.08	2.911	$85+\sigma$	136
Exp't	-6.62 ± 0.04			3.029(3)/3.060(3)	43 ± 5	

is 1.63 MeV), the attractive part of the $^3S_1$ ΛN potential is adjusted to reproduce the correct binding energy $E^{\rm exp}({}^5_\Lambda{\rm He}) = -3.12 \pm 0.02$ MeV. The odd-state ΛN force is assumed to be zero. The partial waves up to $\lambda_{\rm Max} = \ell_{1\,{\rm Max}} = 6$ are included both in the 2α and $\Lambda\alpha$ channels. The direct and exchange Coulomb kernel between two α-clusters is introduced at the nucleon level with a cut-off radius, $R_C = 14$ fm (central case) or 10 fm (ℓs included). Table 1 shows the ground-state (0^+) and the excited-state (2^+) energies of ${}^9_\Lambda$Be, predicted by the SB and other various ΛN potentials used by Hiyama *et al.*[11] In the present calculations using only central forces, the SB potential with the pure Serber character can reproduce the ground-state and excited-state energies within the accuracy of 100 - 200 keV. Table 1 also shows a simple estimate of the ℓs splitting for the $5/2^+$ and $3/2^+$ excited states, due to the spin-orbit interaction predicted by fss2 and FSS.

4. $2\Lambda\alpha$ FADDEEV CALCULATION FOR ${}^{\;\,6}_{\Lambda\Lambda}$He

Next, we use the $\Lambda\alpha$ T-matrix in the $2\alpha\Lambda$ Faddeev calculation, to calculate the ground-state energy of ${}^{\;\,6}_{\Lambda\Lambda}$He. The full coupled-channel T-matrices of fss2 and FSS with the strangeness $S = -2$ and the isospin $I = 0$ [3] are employed for the $\Lambda\Lambda$ RGM T-matrix. We find that the Hiyama's 3-range Gaussian $\Lambda\Lambda$ potential and our old model FSS yield very similar results with the large $\Delta B_{\Lambda\Lambda}$ values, $\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda}({}^{\;\,6}_{\Lambda\Lambda}{\rm He}) - 2B_\Lambda({}^5_\Lambda{\rm He}) \sim 3.6 - 3.7$MeV, since the $\Lambda\Lambda$ phases shifts predicted by both interactions increase up to about 40°. The improved quark model fss2 yields $\Delta B_{\Lambda\Lambda} = 1.41$ MeV. (For the $\Lambda\Lambda$ single-channel T-matrix, this value is reduced to 1.14 MeV.) If we use a simple 2-range Gaussian potential, $V_{\Lambda\Lambda}$(SB), derived from the fss2 $^1S_0$ $\Lambda\Lambda$ phase shift by the supersymmetric inversion method, we obtain $\Delta B_{\Lambda\Lambda} = 1.90$ MeV. We think that the 0.5 MeV difference between the fss2 and $V_{\Lambda\Lambda}$(SB) results is probably because we neglected the coupled-channel effects of the $\Lambda\Lambda\alpha$ channel to the $\Xi N\alpha$ and $\Sigma\Sigma\alpha$ channels. We should also keep in mind that in all of these 3-cluster calculations the Brueckner rearrangement effect [10] of the α-cluster with the magnitude of about -1 MeV (repulsive) is very important. It is further reported in ref. [12] that the quark Pauli effect among the α cluster and the Λ hyperons gives a non-negligible repulsive contribution of 0.1 - 0.2 MeV for the Λ separation energy of ${}^{\;\,6}_{\Lambda\Lambda}$He, even when a rather compact $(3q)$ size of $b = 0.6$ fm is assumed. Taking all of these effects into consideration, we can conclude that the present results by fss2 are in good agreement with the recent experimental value, $\Delta B^{\rm exp}_{\Lambda\Lambda} = 1.01 \pm 0.20$ MeV, [13] deduced from the Nagara event.

5. SUMMARY

The 3-cluster Faddeev formalism using the 2-cluster RGM kernel opens a way to solve few-baryon systems interacting by the quark-model baryon-baryon interactions without spoiling the essential features of the RGM kernel; i.e., non-locality, energy dependence and the existence of the pairwise Pauli-forbidden state. They can also be used for the 3-cluster systems involving α-clusters, like the ${}^{9}_{\Lambda}$Be system. A nice point of this formalism is that the underlying NN and YN interactions are more direcly related to the structure of the hypernuclei than the models assuming simple 2-cluster potentials. In particular, we have found that the most recent quark-model interaction, the model fss2, yields a realistic description of many systems including the triton, hypertriton, ${}^{9}_{\Lambda}$Be and ${}^{6}_{\Lambda\Lambda}$He.

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